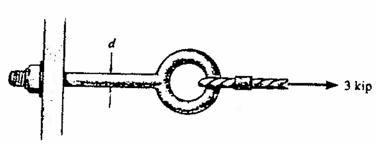
FE Review MofM.



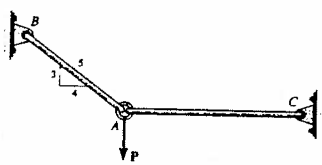
**D–9** The bolt is used to support the load of 3 kip. Determine its diameter $d$ to the nearest $\frac{1}{8}$ in. The allowable normal stress for the bolt is $\sigma_{allow} = 24$ ksi.




Prob. D–9 $d = .399$

**D–10** The two rods support the vertical force of $P = 30$ kN. Determine the diameter of rod $AB$ if the allowable tensile stress for the material is $\sigma_{allow} = 150$ MPa. 20.6

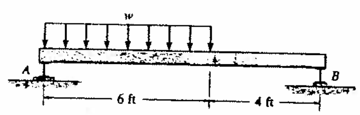
**D–11** The rods $AB$ and $AC$ have diameters of 15 mm and 12 mm, respectively. Determine the largest vertical force **P** that can be applied. The allowable tensile stress for the rods is $\sigma_{allow} = 150$ MPa.




Probs. D–10/D–11

$= 12.7$ KN

**D–12** The allowable bearing stress for the material under the supports $A$ and $B$ is $\sigma_{allow} = 500$ psi. Determine the maximum uniform distributed load $w$ that can be applied to the beam. The bearing plates at $A$ and $B$ have square cross sections of 3 in. × 3 in. and 2 in. × 2 in., respectively.




Prob. D–12

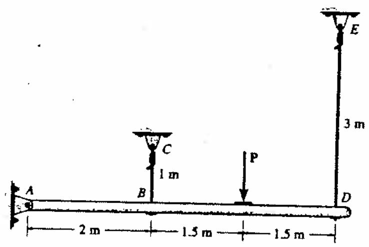
ad B

$w = 1.11 \frac{kip}{ft}$

**Chapter 2—Review All Sections**

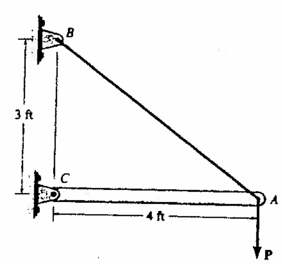
**D–13** A rubber band has an unstretched length of 9 in. If it is stretched around a pole having a diameter of 3 in., determine the average normal strain in the band.

**D–14** The rigid rod is supported by a pin at $A$ and wires $BC$ and $DE$. If the maximum allowable normal strain in each wire is $\epsilon_{allow} = 0.003$, determine the maximum vertical displacement of the load **P**. $\delta = 5.25$ mm




Prob. D–14

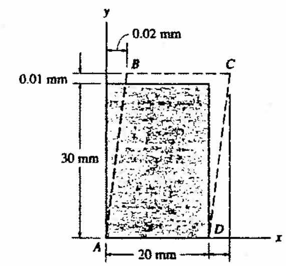
**D–15** The load **P** causes a normal strain of 0.0045 in./in. in cable $AB$. Determine the angle of rotation of the rigid beam due to the loading if the beam is originally horizontal before it is loaded.




Prob. D–15



**D–16** The square piece of material is deformed into the dashed position. Determine the shear strain at corner $C$.

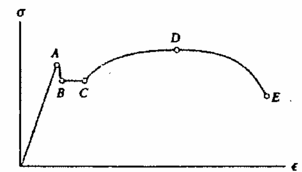



Prob. D–16 $\gamma_C = 0.666 \times 10^{-3}$

**Chapter 3—Review Sections 3.1–3.7**

**D–17** Define homogeneous material.

**D–18** Indicate the points on the stress–strain diagram which represent the proportional limit and the ultimate stress.




Prob. D–18

**D–19** Define the modulus of elasticity $E$.

**D–20** At room temperature, mild steel is a ductile material. True or false.

T

**D–21** Engineering stress and strain are calculated using the *actual* cross-sectional area and length of the specimen. True or false.

F

**D–22** If a rod is subjected to an axial load, there is only strain in the material in the direction of the load. True or false.

F

**D–23** A 100-mm long rod has a diameter of 15 mm. If an axial tensile load of 100 kN is applied, determine its change in length. $E = 200$ GPa. 0.283 mm

**D–24** A bar has a length of 8 in. and cross-sectional area of 12 in$^2$. Determine the modulus of elasticity of the material if it is subjected to an axial tensile load of 10 kip and stretches 0.003 in. The material has linear-elastic behavior. $2.22 \times 10^6$ psi

**D–25** A 10-mm-diameter brass rod has a modulus of elasticity of $E = 100$ GPa. If it is 4 m long and subjected to an axial tensile load of 6 kN, determine its elongation. 3.06 mm

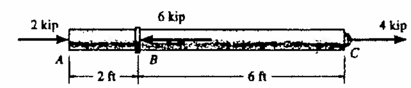
**D–26** A 100-mm long rod has a diameter of 15 mm. If an axial tensile load of 10 kN is applied to it, determine its change in diameter. $E = 70$ GPa, $\nu = 0.35$.

**Chapter 4—Review Sections 4.1–4.6**

**D–27** What is Saint-Venant's principle?

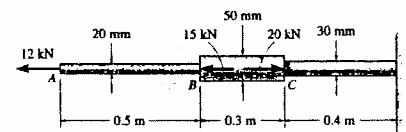
**D–28** What are the two conditions for which the principle of superposition is valid?

**D–29** Determine the displacement of end $A$ with respect to end $C$ of the shaft. The cross-sectional area is 0.5 in$^2$ and $E = 29(10^3)$ ksi. 0.0166 in

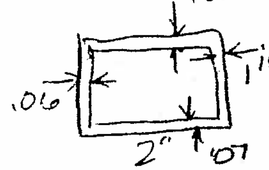



Prob. D–29

**D–30** Determine the displacement of end $A$ with respect to $C$ of the shaft. The diameters of each segment are indicated in the figure. $E = 200$ GPa.




Prob. D–30 0.116



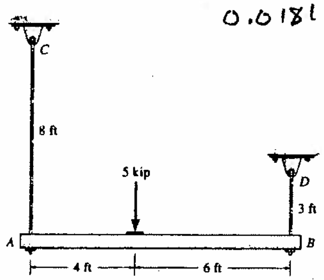
$\tau = \frac{T}{2At} =$

Also descuss thin walled Hollow sections



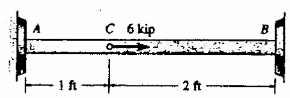
**D–31** Determine the angle of tilt of the rigid beam when it is subjected to the load of 5 kip. Before the load is applied the beam is horizontal. Each rod has a diameter of 0.5 in., and $E = 29(10^3)$ ksi.

0.0181°




Prob. D–32

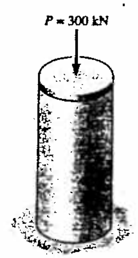
**D–32** The uniform bar is subjected to the load of 6 kip. Determine the horizontal reactions at the supports $A$ and $B$.




$F_A = 4$
$F_B = 2$

Prob. D–32

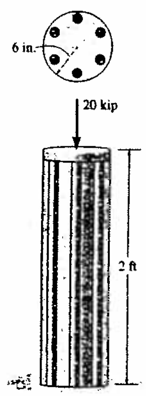
**D–33** The cylinder is made from steel and has an aluminum core. If its ends are subjected to the axial force of 300 kN, determine the average normal stress in the steel. The cylinder has an outer diameter of 100 mm and an inner diameter of 80 mm. $E_{st} = 200$ GPa, $E_{al} = 73.1$ GPa.




$\sigma_{st} = 64.3$ mm

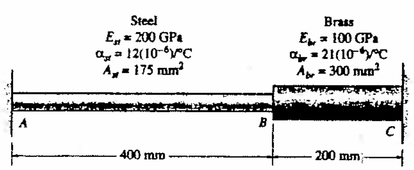
Prob. D–33

**D–34** The column is constructed from concrete and six steel reinforcing rods. If it is subjected to an axial force of 20 kip, determine the force supported by the concrete. Each rod has a diameter of 0.75 in. $E_{conc} = 4.20(10^3)$ ksi, $E_{st} = 29(10^3)$ ksi.




Prob. D–34

**D–35** Two bars, each made of a different material, are connected and placed between two walls when the temperature is $T_1 = 15°C$. Determine the force exerted on the (rigid) supports when the temperature becomes $T_2 = 25°C$. The material properties and cross-sectional area of each bar are given in the figure.

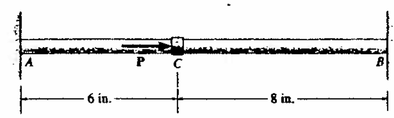



Prob. D–35

F = 4.96 kN



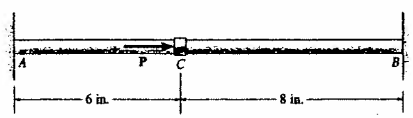
**D–36** The aluminum rod has a diameter of 0.5 in. and is attached to the rigid supports at $A$ and $B$ when $T_1 = 80°F$. If the temperature becomes $T_2 = 100°F$, and an axial force of $P = 1200$ lb is applied to the rigid collar as shown, determine the reactions at $A$ and $B$. $\alpha_{al} = 12.8(10^{-6})/°F$, $E_{al} = 10.6(10^6)$ psi.




Prob. D–36

$F_A = 153$ lb

**D–37** The aluminum rod has a diameter of 0.5 in. and is attached to the rigid supports at $A$ and $B$ when $T_1 = 80°F$. Determine the force **P** that must be applied to the rigid collar so that, when $T_2 = 50°F$, the reaction at $B$ is zero. $\alpha_{al} = 12.8(10^{-6})/°F$, $E_{al} = 10.6(10^3)$ ksi.

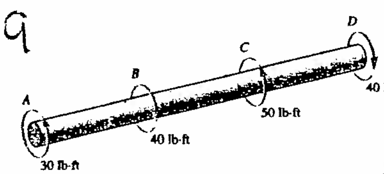



Prob. D–37

### Chapter 5—Review Sections 5.1–5.5

**D–38** Can the torsion formula, $\tau = Tc/J$, be used if the cross section is noncircular?

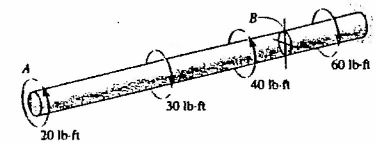
**D–39** The solid 0.75-in.-diameter shaft is used to transmit the torques shown. Determine the absolute maximum shear stress developed in the shaft.




Prob. D–39

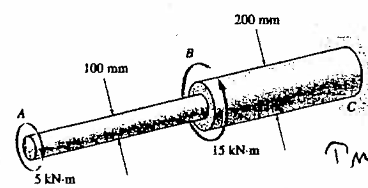
$\tau_{max} = 5.79$ ksi

**D–40** The solid 1.5-in.-diameter shaft is used to transmit the torques shown. Determine the shear stress developed in the shaft at point $B$.




Prob. D–40

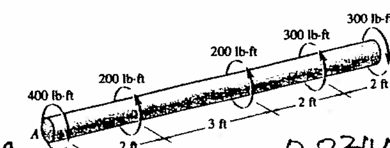
**D–41** The solid shaft is used to transmit the torques shown. Determine the absolute maximum shear stress developed in the shaft.




$\tau_{max} = 6.37$ mpa

Prob. D–41

**D–42** The shaft is subjected to the torques shown. Determine the angle of twist of end $A$ with respect to end $B$. The shaft has a diameter of 1.5 in. $G = 11(10^3)$ ksi.

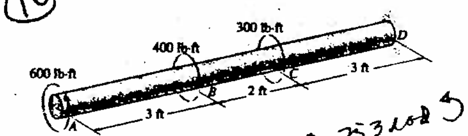



0.021 rad.

Prob. D–42

**D–43** Determine the angle of twist of the 1-in.-diameter shaft at end $A$ when it is subjected to the torsional loading shown. $G = 11(10^3)$ ksi.

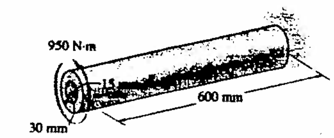
(10)




Prob. D–43

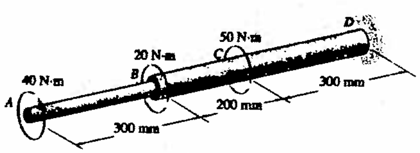
0.753 rad

**D–47** The shaft is made from a steel tube having a brass core. If it is fixed to the rigid support, determine the angle of twist that occurs at its end. $G_{st} = 75$ GPa and $G_{br} = 37$ GPa.



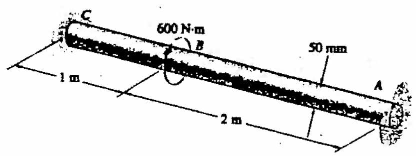

Prob. D–47

**D–44** The shaft consists of a solid section $AB$ with a diameter of 30 mm, and a tube $BD$ with an inner diameter of 25 mm and outer diameter of 50 mm. Determine the angle of twist at its end $A$ when it is subjected to the torsional loading shown. $G = 75$ GPa.




Prob. D–44

**D–48** Determine the absolute maximum shear stress in the shaft. $JG$ is constant.




Prob. D–48

$\tau_{max} = 16.3$ MPa

**D–45** A motor delivers 200 hp to a steel shaft, which is tubular and has an outer diameter of 1.75 in. If it is rotating at 150 rad/s, determine its largest inner diameter to the nearest $\frac{1}{8}$ in. if the allowable shear stress for the material is $\tau_{allow} = 20$ ksi.

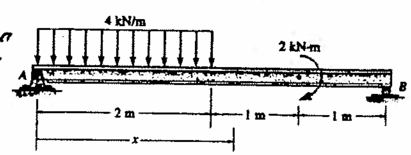
$d_i = 1.625$ in $= 1\frac{5}{8}$"

**D–46** A motor delivers 300 hp to a steel shaft, which is tubular and has an outer diameter of 2.5 in. and an inner diameter of 2 in. Determine the smallest angular velocity at which it can rotate if the allowable shear stress for the material is $\tau_{allow} = 20$ ksi.

$\omega = 54.7$ rad/s

**Chapter 6—Review Sections 6.1–6.5**

**D–49** Determine the internal moment in the beam as a function of $x$, where $2 \text{ m} \leq x < 3$ m.

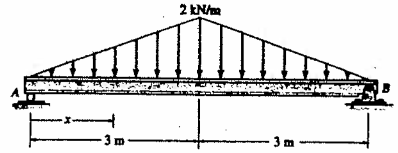



Prob. D–49

$M = 8 - 2.5x$



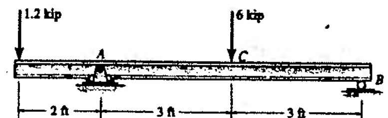
**D–50** Determine the internal moment in the beam as a function of $x$, where $0 \leq x \leq 3$ m.




Prob. D–50

11

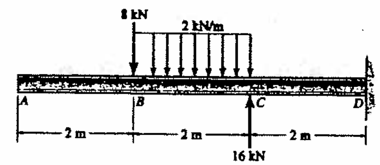
**D–51** Determine the maximum moment in the beam.




Prob. D–51

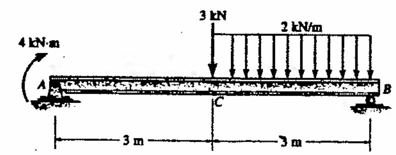
$M_{max} = 7.80$ k·ft

**D–52** Determine the absolute maximum bending stress in the beam.




Prob. D–52

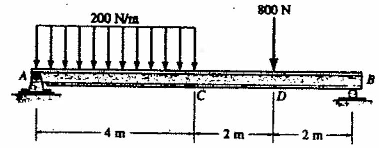
**D–53** Determine the maximum moment in the beam.




Prob. D–53

$M_{max} = 20$ kN

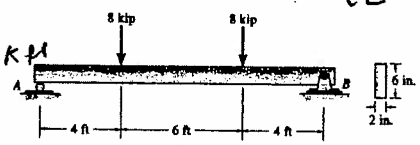
**D–54** Determine the maximum moment in the beam.




Prob. D–54

**D–55** Determine the absolute maximum bending stress in the beam.

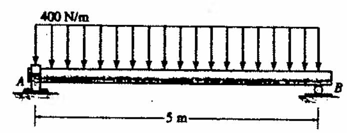
12




Prob. D–55

37 ksi

**D–56** Determine the maximum bending stress in the 50-mm-diameter rod at $C$.

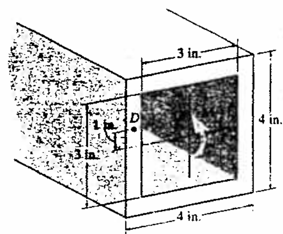



Prob. D–56

102 MPa

**D–57** What is the strain in a beam at the neutral axis?

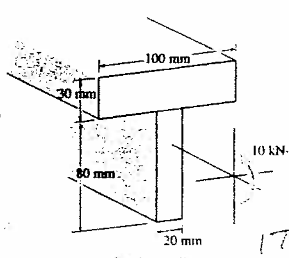
**D–58** Determine the moment $M$ that should be applied to the beam in order to create a compressive stress at point $D$ of 10 ksi.




Prob. D–58

**D–59** Determine the maximum bending stress in the beam.

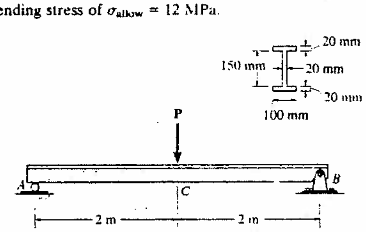
13




179 MPa

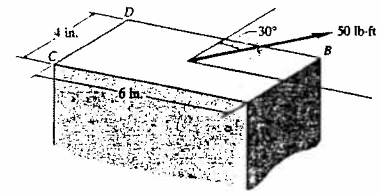
Prob. D–59

**D–60** Determine the maximum load $P$ that can be applied to the beam that is made from a material having an allowable bending stress of $\sigma_{allow} = 12$ MPa.




Prob. D–60

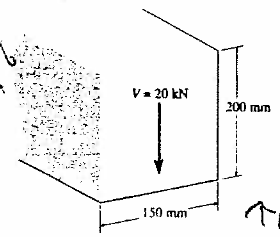
**D–61** Determine the maximum stress in the beam's cross section.




Prob. D–61

Chapter 7—Review Sections 7.1–7.4

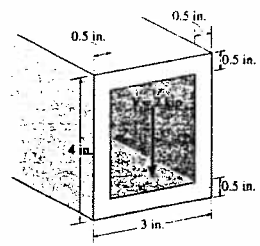
**D–62** Determine the maximum shear stress in the beam.




$\tau_{max} = 1$ MPa

Prob. D–62

**D–63** The beam has a rectangular cross section and is subjected to a shear of $V = 2$ kN. Determine the maximum shear stress in the beam.

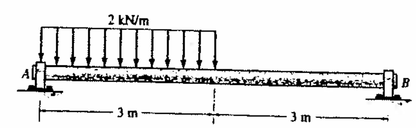



Prob. D–63

$\tau_{max} = 0.552$ ksi

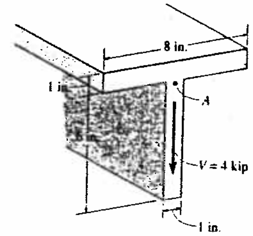


**D–64** Determine the absolute maximum shear stress in the shaft having a diameter of 60 mm.




Prob. D–64

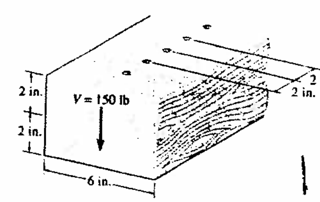
**D–65** Determine the shear stress in the beam at point $A$, which is located at the top of the web.




Prob. D–65

$\tau = 79$ psi

**D–66** The beam is made from two boards fastened together at the top and bottom with nails spaced every 2 in. If an internal shear force of $V = 150$ lb is applied to the boards, determine the shear force resisted by each nail.

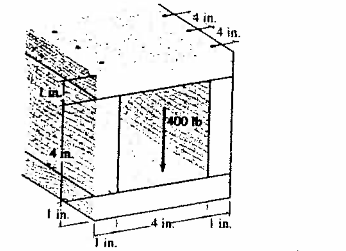



14

Prob. D–66

112.5 lb

**D–67** The beam is made from four boards fastened together at the top and bottom with two rows of nails spaced every 4 in. If an internal shear force of $V = 400$ lb is applied to the boards, determine the shear force resisted by each nail.




Prob. D–67

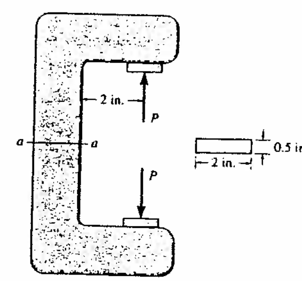
138 lb

Chapter 8—Review 8.1 Sections

**D–68** A cylindrical tank is subjected to an internal pressure of 80 psi. If the internal diameter of the tank is 30 in., and the wall thickness is 0.3 in., determine the maximum normal stress in the material.

**D–69** A pressurized spherical tank is to be made of 0.25-in.-thick steel. If it is subjected to an internal pressure of $p = 150$ psi, determine its inner diameter if the maximum normal stress is not to exceed 10 ksi.

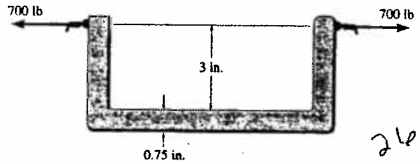
**D–70** Determine the magnitude of the load $P$ that will cause a maximum normal stress of $\sigma_{max} = 30$ ksi in the link along section $a$–$a$.




Prob. D–70

P = 3 kips

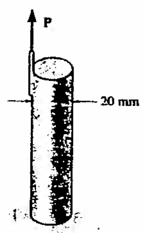
**D–71** Determine the maximum normal stress in the horizontal portion of the bracket. The bracket has a thickness of 1 in. and a width of 0.75 in.




Prob. D–71

26.1 ksi

**D–72** Determine the maximum load $P$ that can be applied to the rod so that the normal stress in the rod does not exceed $\sigma_{max} = 30$ MPa.

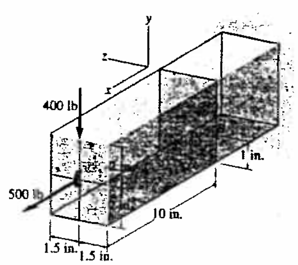



15

1.88 kN

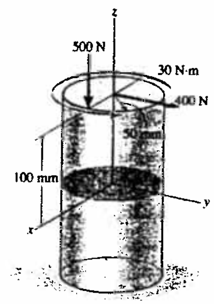
Prob. D–72

**D–73** The beam has a rectangular cross section and is subjected to the loading shown. Determine the components of stress $\sigma_x$, $\sigma_y$, and $\tau_{xy}$ at point $B$.




Prob. D–73

**D–74** The solid cylinder is subjected to the loading shown. Determine the components of stress at point $B$.



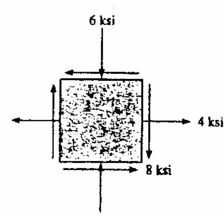

Prob. D–74

**Chapter 9—Review Sections 9.1–9.3**

**D–75** When the state of stress at a point is represented by the principal stress, no shear stress will act on the element. True or false.

**D–76** The state of stress at a point is shown on the element. Determine the maximum principal stress.

16



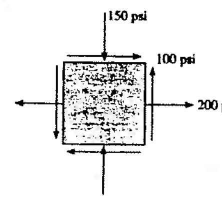

Prob. D–76

$\sigma_1 = 8.43$ ksi

$\sigma_2 = -10.4$ ksi



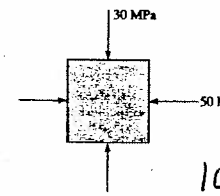
**D–77** The state of stress at a point is shown on the element. Determine the maximum in-plane shear stress.




Prob. D–77

$\tau_{max\ in\text{-}plane} = 202$ psi

**D–78** The state of stress at a point is shown on the element. Determine the maximum in-plane shear stress.

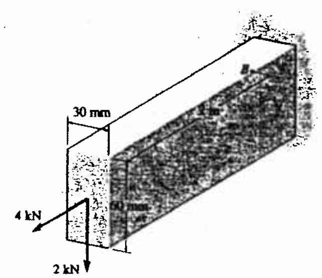



17

10 MPa

Prob. D–78

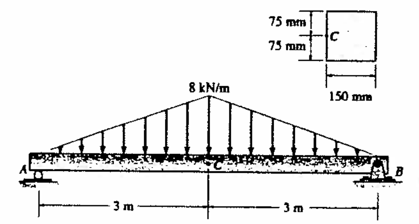
**D–79** The beam is subjected to the load at its end. Determine the maximum principal stress at point $B$.




Prob. D–79

$\sigma_B = 224$ MPa

**D–80** The beam is subjected to the loading shown. Determine the principal stress at point $C$.

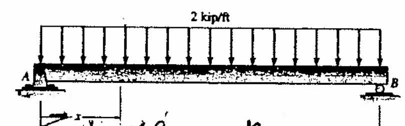



Prob. D–80

**Chapter 12—Review Sections 12.1–12.2, 12.5**

**D–81** The beam is subjected to the loading shown. Determine the equation of the elastic curve. $EI$ is constant.

18

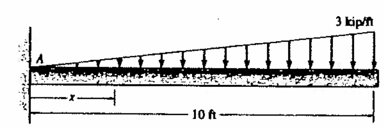



Likely ask for part.

Prob. D–81

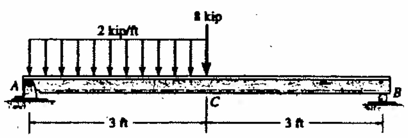
$v = \frac{1}{EI}\left[-\frac{1}{12}x^4 + .5x^3 - 2.25x\right)$

**D–82** The beam is subjected to the loading shown. Determine the equation of the elastic curve. $EI$ is constant.



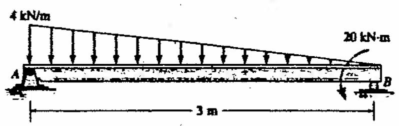

Prob. D–82

**D–83** Determine the displacement at point $C$ of the beam shown. Use the method of superposition. $EI$ is constant.




Prob. D–83

**D–84** Determine the slope at point $A$ of the beam shown. Use the method of superposition. $EI$ is constant.



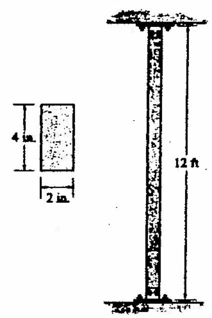

Prob. D–84

**Chapter 13—Review Sections 13.1–13.3**

**D–85** The critical load is the maximum axial load that a column can support when it is on the verge of buckling. This loading represents a case of neutral equilibrium. True or false.

**D–86** A 50-in.-long rod is made from a 1-in.-diameter steel rod. Determine the critical buckling load if the ends are fixed supported. $E = 29(10^3)$ ksi, $\sigma_Y = 36$ ksi.

**D–87** A 12-ft wooden rectangular column has the dimensions shown. Determine the critical load if the ends are assumed to be pin-connected. $E = 1.6(10^3)$ ksi. Yielding does not occur.




Prob. D–87

解
703 kip

**D–88** A steel pipe is fixed-supported at its ends. If it is 5 m long and has an outer diameter of 50 mm and a thickness of 10 mm, determine the maximum axial load $P$ that it can carry without buckling. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.

**D–89** A steel pipe is pin-supported at its ends. If it is 6 ft long and has an outer diameter of 2 in., determine its smallest thickness so that it can support an axial load of $P = 40$ kip without buckling. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 36$ ksi.

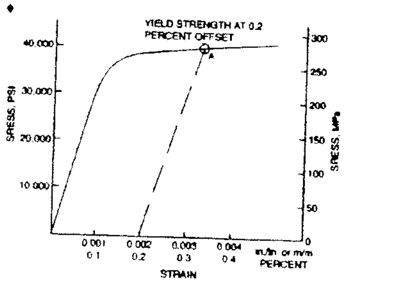
**D–90** Determine the smallest diameter of a solid 40-in.-long steel rod, to the nearest $\frac{1}{16}$ in., that will support an axial load of $P = 3$ kip without buckling. The ends are pin connected. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 36$ ksi.

# MECHANICS OF MATERIALS

## UNIAXIAL STRESS-STRAIN

### Stress-Strain Curve for Mild Steel

♦




The slope of the linear portion of the curve equals the modulus of elasticity

### DEFINITIONS

**Engineering Strain**

$\varepsilon = \Delta L / L_o$, where

$\varepsilon$ = engineering strain (units per unit),

$\Delta L$ = change in length (units) of member,

$L_o$ = original length (units) of member.

**Percent Elongation**

$$\%\ \text{Elongation} = \left(\frac{\Delta L}{L_o}\right) \times 100$$

**Percent Reduction in Area (RA)**

The % reduction in area from initial area, $A_i$, to final area, $A_f$, is:

$$\%\text{RA} = \left(\frac{A_i - A_f}{A_i}\right) \times 100$$

**True Stress is load divided by actual cross-sectional area.**

**Shear Stress-Strain**

$\gamma = \tau/G$, where

$\gamma$ = shear strain,

$\tau$ = shear stress, and

$G$ = *shear modulus* (constant in linear force-deformation relationship)

$$G = \frac{E}{2(1+v)}, \text{ where}$$

$E$ = modulus of elasticity

$v$ = *Poisson's ratio*, and

= – (lateral strain)/(longitudinal strain).

**Uniaxial Loading and Deformation**

$\sigma = P/A$, where

$\sigma$ = stress on the cross section,

$P$ = loading, and

$A$ = cross-sectional area.

$\varepsilon = \delta/L$, where

$\delta$ = elastic longitudinal deformation and

$L$ = length of member.

$$E = \sigma/\varepsilon = \frac{P/A}{\delta/L}$$

$$\delta = \frac{PL}{AE}$$

### THERMAL DEFORMATIONS

$\delta_t = \alpha L (T - T_o)$, where

$\delta_t$ = deformation caused by a change in temperature,

$\alpha$ = temperature coefficient of expansion,

$L$ = length of member,

$T$ = final temperature, and

$T_o$ = initial temperature.

### CYLINDRICAL PRESSURE VESSEL

**Cylindrical Pressure Vessel**

For internal pressure only, the stresses at the inside wall are:

$$\sigma_t = P_i \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} \quad \text{and} \quad 0 > \sigma_r > -P_i$$

For external pressure only, the stresses at the outside wall are:

$$\sigma_t = -P_o \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} \quad \text{and} \quad 0 > \sigma_r > -P_o, \text{ where}$$

$\sigma_t$ = tangential (hoop) stress,

$\sigma_r$ = radial stress,

$P_i$ = internal pressure,

$P_o$ = external pressure,

$r_i$ = inside radius, and

$r_o$ = outside radius

For vessels with end caps, the axial stress is:

$$\sigma_a = P_i \frac{r_i^2}{r_o^2 - r_i^2}$$

These are principal stresses.

♦ Flinn, Richard A. & Paul K. Trojan, *Engineering Materials & Their Applications*, 4th ed., Houghton Mifflin Co., 1990.



When the thickness of the cylinder wall is about one-tenth or less, of inside radius, the cylinder can be considered as thin-walled. In which case, the internal pressure is resisted by the hoop stress and the axial stress.

$$\sigma_t = \frac{P_i r}{t} \quad and \quad \sigma_a = \frac{P_i r}{2t}$$

where $t$ = wall thickness.

## STRESS AND STRAIN

**Principal Stresses**

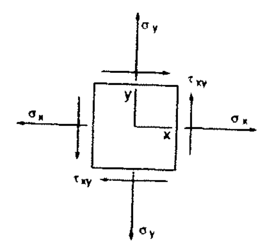
For the special case of a *two-dimensional* stress state, the equations for principal stress reduce to

$$\sigma_a, \sigma_b = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_c = 0$$

The two nonzero values calculated from this equation are temporarily labeled $\sigma_a$ and $\sigma_b$ and the third value $\sigma_c$ is always zero in this case. Depending on their values, the three roots are then labeled according to the convention: *algebraically largest* = $\sigma_1$, *algebraically smallest* = $\sigma_3$, *other* = $\sigma_2$. A typical 2D stress element is shown below with all indicated components shown in their positive sense.

♦




**Mohr's Circle – Stress, 2D**

To construct a Mohr's circle, the following sign conventions are used.

1. Tensile normal stress components are plotted on the horizontal axis and are considered positive. Compressive normal stress components are negative.
2. For constructing Mohr's circle only, shearing stresses are plotted above the normal stress axis when the pair of shearing stresses, acting on opposite and parallel faces of an element, forms a clockwise couple. Shearing stresses are plotted below the normal axis when the shear stresses form a counterclockwise couple.



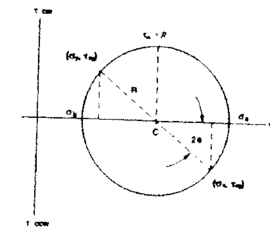
The circle drawn with the center on the normal stress (horizontal) axis with center, $C$, and radius, $R$, where

$$C = \frac{\sigma_x + \sigma_y}{2}, \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

The two nonzero principal stresses are then:

♦ $\sigma_a = C + R$

$\sigma_b = C - R$




The maximum *inplane* shear stress is $\tau_{in} = R$. However, the maximum shear stress considering three dimensions is always

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

**Hooke's Law**

Three-dimensional case:

$\varepsilon_x = (1/E)[\sigma_x - v(\sigma_y + \sigma_z)]$ $\qquad \gamma_{xy} = \tau_{xy}/G$

$\varepsilon_y = (1/E)[\sigma_y - v(\sigma_z + \sigma_x)]$ $\qquad \gamma_{yz} = \tau_{yz}/G$

$\varepsilon_z = (1/E)[\sigma_z - v(\sigma_x + \sigma_y)]$ $\qquad \gamma_{zx} = \tau_{zx}/G$

Plane stress case ($\sigma_z = 0$):

$\varepsilon_x = (1/E)(\sigma_x - v\sigma_y)$

$\varepsilon_y = (1/E)(\sigma_y - v\sigma_x)$

$\varepsilon_z = -(1/E)(v\sigma_x + v\sigma_y)$

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \frac{E}{1-v^2}\begin{bmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & \frac{1-v}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}$$

Uniaxial case ($\sigma_y = \sigma_z = 0$): $\qquad \sigma_x = E\varepsilon_x$ or $\sigma = E\varepsilon$, where

$\varepsilon_x, \varepsilon_y, \varepsilon_z$ = normal strain,

$\sigma_x, \sigma_y, \sigma_z$ = normal stress,

$\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$ = shear strain,

$\tau_{xy}, \tau_{yz}, \tau_{zx}$ = shear stress,

$E$ = modulus of elasticity,

$G$ = shear modulus, and

$v$ = Poisson's ratio

♦ Crandall, S.H. & N.C. Dahl, *An Introduction to The Mechanics of Solids*, McGraw-Hill Book Co., Inc., 1959.

## STATIC LOADING FAILURE THEORIES

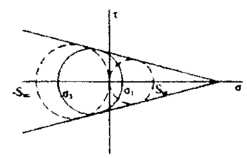
### Brittle Materials

#### Maximum-Normal-Stress Theory

The maximum-normal-stress theory states that failure occurs when one of the three principal stresses equals the strength of the material. If $\sigma_1 \ge \sigma_2 \ge \sigma_3$, then the theory predicts that failure occurs whenever $\sigma_1 \ge S_{ut}$ or $\sigma_3 \le -S_{uc}$ where $S_{ut}$ and $S_{uc}$ are the tensile and compressive strengths, respectively.

#### Coulomb-Mohr Theory

The Coulomb-Mohr theory is based upon the results of tensile and compression tests. On the σ, τ coordinate system, one circle is plotted for $S_{ut}$ and one for $S_{uc}$. As shown in the figure, lines are then drawn tangent to these circles. The Coulomb-Mohr theory then states that fracture will occur for any stress situation that produces a circle that is either tangent to or crosses the envelope defined by the lines tangent to the $S_{ut}$ and $S_{uc}$ circles.




If $\sigma_1 \ge \sigma_2 \ge \sigma_3$ and $\sigma_3 < 0$, then the theory predicts that yielding will occur whenever

$$\frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}} \ge 1$$

### Ductile Materials

#### Maximum-Shear-Stress Theory

The maximum-shear-stress theory states that yielding begins when the maximum shear stress equals the maximum shear stress in a tension-test specimen of the same material when that specimen begins to yield. If $\sigma_1 \ge \sigma_2 \ge \sigma_3$, then the theory predicts that yielding will occur whenever $\tau_{max} \ge S_y/2$ where $S_y$ is the yield strength.

#### Distortion-Energy Theory

The distortion-energy theory states that yielding begins whenever the distortion energy in a unit volume equals the distortion energy in the same volume when uniaxially stressed to the yield strength. The theory predicts that yielding will occur whenever

$$\left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}{2}\right]^{1/2} \ge S_y$$

The term on the left side of the inequality is known as the effective or Von Mises stress. For a biaxial stress state the effective stress becomes

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}$$

or

$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$$

where $\sigma_A$ and $\sigma_B$ are the two nonzero principal stresses and $\sigma_x$, $\sigma_y$, and $\tau_{xy}$ are the stresses in orthogonal directions.

## VARIABLE LOADING FAILURE THEORIES

Modified Goodman Theory: The modified Goodman criterion states that a fatigue failure will occur whenever

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} \ge 1 \quad \text{or} \quad \frac{\sigma_{max}}{S_y} \ge 1, \qquad \sigma_m \ge 0,$$

where

$S_e$ = fatigue strength,
$S_{ut}$ = ultimate strength,
$S_y$ = yield strength,
$\sigma_a$ = alternating stress, and
$\sigma_m$ = mean stress.
$\sigma_{max} = \sigma_m + \sigma_a$

Soderberg Theory: The Soderberg theory states that a fatigue failure will occur whenever

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} \ge 1, \qquad \sigma_m \ge 0$$

Endurance Limit for Steels: When test data is unavailable, the endurance limit for steels may be estimated as

$$S_e' = \begin{cases} 0.5\,S_{ut},\ S_{ut} \le 1{,}400\text{ MPa} \\ 700\text{ MPa},\ S_{ut} > 1{,}400\text{ MPa} \end{cases}$$





Endurance Limit Modifying Factors: Endurance limit modifying factors are used to account for the differences between the endurance limit as determined from a rotating beam test, $S_e'$, and that which would result in the real part, $S_e$.

$$S_e = k_a k_b k_c k_d k_e S_e'$$

where

*Surface Factor*, $k_a = aS_{ut}^b$

| Surface Finish | Factor *a* | | Exponent *b* |
|---|---|---|---|
| | kpsi | MPa | |
| Ground | 1.34 | 1.58 | –0.085 |
| Machined or CD | 2.70 | 4.51 | –0.265 |
| Hot rolled | 14.4 | 57.7 | –0.718 |
| As forged | 39.9 | 272.0 | –0.995 |

*Size Factor*, $k_b$:

For bending and torsion:

$d \le 8$ mm; $k_b = 1$

8 mm $\le d \le 250$ mm; $k_b = 1.189 d_{eff}^{-0.097}$

$d > 250$ mm; $0.6 \le k_b \le 0.75$

For axial loading: $k_b = 1$

*Load Factor*, $k_c$:

$k_c = 0.923$ axial loading, $S_{ut} \le 1{,}520$ MPa

$k_c = 1$ axial loading, $S_{ut} > 1{,}520$ MPa

$k_c = 1$ bending

*Temperature Factor*, $k_d$:

for T ≤ 450°C, $k_d = 1$

*Miscellaneous Effects Factor*, $k_e$: Used to account for strength reduction effects such as corrosion, plating, and residual stresses. In the absence of known effects, use $k_e = 1$.

## TORSION

Torsion stress in circular solid or thick-walled ($t > 0.1\ r$) shafts:

$$\tau = \frac{Tr}{J}$$

where $J$ = polar moment of inertia (see table at end of **DYNAMICS** section).

## TORSIONAL STRAIN

$$\gamma_{\phi z} = \lim_{\Delta z \to 0} r(\Delta\phi/\Delta z) = r(d\phi/dz)$$

The shear strain varies in direct proportion to the radius, from zero strain at the center to the greatest strain at the outside of the shaft. $d\phi/dz$ is the twist per unit length or the rate of twist.

$$\tau_{\phi z} = G\,\gamma_{\phi z} = Gr\,(d\phi/dz)$$

$$T = G\,(d\phi/dz)\int_A r^2 dA = GJ(d\phi/dz)$$

$$\phi = \int_0^L \frac{T}{GJ}\,dz = \frac{TL}{GJ}, \text{ where}$$

$\phi$ = total angle (radians) of twist,
$T$ = torque, and
$L$ = length of shaft.

$T/\phi$ gives the *twisting moment per radian of twist*. This is called the *torsional stiffness* and is often denoted by the symbol $k$ or $c$.

### For Hollow, Thin-Walled Shafts

$$\tau = \frac{T}{2A_m t}, \text{ where}$$

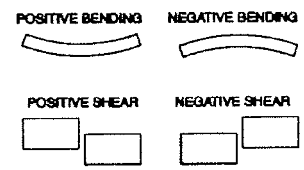
$t$ = thickness of shaft wall and
$A_m$ = the total mean area enclosed by the shaft measured to the midpoint of the wall.

## BEAMS

### Shearing Force and Bending Moment Sign Conventions

1. The bending moment is *positive* if it produces bending of the beam *concave upward* (compression in top fibers and tension in bottom fibers).
2. The shearing force is *positive* if the *right portion of the beam tends to shear downward with respect to the left*.

•




The relationship between the load ($q$), shear ($V$), and moment ($M$) equations are:

$$q(x) = -\frac{dV(x)}{dx}$$

$$V = \frac{dM(x)}{dx}$$

$$V_2 - V_1 = \int_{x_1}^{x_2}\left[-q(x)\right]dx$$

$$M_2 - M_1 = \int_{x_1}^{x_2} V(x)\,dx$$

### Stresses in Beams

$\varepsilon_x = -y/\rho$, where

$\rho$ = the radius of curvature of the deflected axis of the beam, and
$y$ = the distance from the neutral axis to the longitudinal fiber in question.

• Timoshenko, S. and Gleason H. MacCullough, *Elements of Strength of Materials*, K. Van Nostrand Co./Wadsworth Publishing Co., 1949.

Using the stress-strain relationship $\sigma = E\varepsilon$,

Axial Stress: $\sigma_x = -Ey/\rho$, where

$\sigma_x$ = the normal stress of the fiber located $y$-distance from the neutral axis.

$1/\rho = M/(EI)$, where

$M$ = the moment at the section and

$I$ = the *moment of inertia* of the cross-section.

$\sigma_x = -My/I$, where

$y$ = the distance from the neutral axis to the fiber location above or below the axis. Let $y = c$, where $c$ = distance from the neutral axis to the outermost fiber of a symmetrical beam section.

$\sigma_x = \pm Mc/I$

Let $S = I/c$: then, $\sigma_x = \pm M/S$, where

$S$ = the *elastic section modulus* of the beam member.

Transverse shear flow: $q = VQ/I$ and

Transverse shear stress: $\tau_{xy} = VQ/(Ib)$, where

$q$ = shear flow,

$\tau_{xy}$ = shear stress on the surface,

$V$ = shear force at the section,

$b$ = width or thickness of the cross-section, and

$Q$ = $A'\bar{y}'$, where

$A'$ = area above the layer (or plane) upon which the desired transverse shear stress acts and

$\bar{y}'$ = distance from neutral axis to area centroid.

### Deflection of Beams

Using $1/\rho = M/(EI)$,

$EI\dfrac{d^2y}{dx^2} = M$, differential equation of deflection curve

$EI\dfrac{d^3y}{dx^3} = dM(x)/dx = V$

$EI\dfrac{d^4y}{dx^4} = dV(x)/dx = -q$

Determine the deflection curve equation by double integration (apply boundary conditions applicable to the deflection and/or slope).

$EI\,(dy/dx) = \int M(x)\,dx$

$EIy = \int[\int M(x)\,dx]\,dx$

The constants of integration can be determined from the physical geometry of the beam.

## COLUMNS

For long columns with pinned ends:

Euler's Formula

$$P_{cr} = \frac{\pi^2 EI}{\ell^2}, \text{ where}$$

$P_{cr}$ = critical axial loading,

$\ell$ = unbraced column length.

substitute $I = r^2A$:

$$\frac{P_{cr}}{A} = \frac{\pi^2 E}{(\ell/r)^2}, \text{ where}$$

$r$ = *radius of gyration* and

$\ell/r$ = *slenderness ratio* for the column.

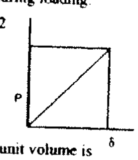
For further column design theory, see the **CIVIL ENGINEERING and MECHANICAL ENGINEERING** sections.

## ELASTIC STRAIN ENERGY

If the strain remains within the elastic limit, the work done during deflection (extension) of a member will be transformed into potential energy and can be recovered.

If the final load is $P$ and the corresponding elongation of a tension member is $\delta$, then the total energy $U$ stored is equal to the work $W$ done during loading.

$U = W = P\delta/2$




The strain energy per unit volume is

$u = U/AL = \sigma^2/2E$ (for tension)

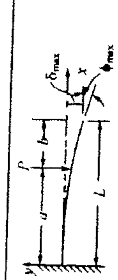
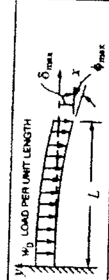
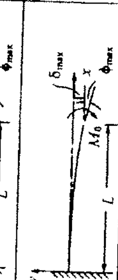
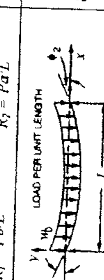
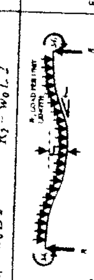
## MATERIAL PROPERTIES

| Material | Units | Steel | Aluminum | Cast Iron | Wood (Fir) |
|---|---|---|---|---|---|
| **Modulus of Elasticity, E** | Mpsi | 29.0 | 10.0 | 14.5 | 1.6 |
| | GPa | 200.0 | 69.0 | 100.0 | 11.0 |
| **Modulus of Rigidity, G** | Mpsi | 11.5 | 3.8 | 6.0 | 0.6 |
| | GPa | 80.0 | 26.0 | 41.4 | 4.1 |
| **Poisson's Ratio, ν** | | 0.30 | 0.33 | 0.21 | 0.33 |
| **Coefficient of Thermal Expansion, α** | $10^{-6}/°F$ | 6.5 | 13.1 | 6.7 | 1.7 |
| | $10^{-6}/°C$ | 11.7 | 23.6 | 12.1 | 3.0 |





**Beam Deflection Formulas – Special Cases**
**(δ is positive downward)**

| | | | |
|---|---|---|---|
|  | $\delta = \frac{Pa^2}{6EI}(3x - a)$, for $x > a$<br>$\delta = \frac{Px^2}{6EI}(-x + 3a)$, for $x \le a$ | $\delta_{max} = \frac{Pa^2}{6EI}(3L - a)$ | $\phi_{max} = \frac{Pa^2}{2EI}$ |
|  | $\delta = \frac{w_o x^2}{24EI}(x^2 + 6L^2 - 4Lx)$ | $\delta_{max} = \frac{w_o L^4}{8EI}$ | $\phi_{max} = \frac{w_o L^3}{6EI}$ |
|  | $\delta = \frac{M_o x^2}{2EI}$ | $\delta_{max} = \frac{M_o L^2}{2EI}$ | $\phi_{max} = \frac{M_o L}{EI}$ |
|  $R_1 = Pb/L$  $R_2 = Pa/L$ | $\delta = \frac{Pb}{6LEI}\left[\frac{L}{b}(x-a)^3 - x^3 + (L^2 - b^2)x\right]$, for $x > a$<br>$\delta = \frac{Pb}{6LEI}\left[-x^3 + (L^2 - b^2)x\right]$, for $x \le a$ | $\delta_{max} = \frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}LEI}$ at $x = \sqrt{\frac{L^2 - b^2}{3}}$ | $\phi_1 = \frac{Pab(2L - a)}{6LEI}$<br>$\phi_2 = \frac{Pab(2L - b)}{6LEI}$ |
|  $R_1 = w_o L/2$  $R_2 = w_o L/2$ | $\delta = \frac{w_o x}{24EI}(L^3 - 2Lx^2 + x^3)$ | $\delta_{max} = \frac{5w_o L^4}{384EI}$ | $\phi_1 = \phi_2 = \frac{w_o L^3}{24EI}$ |
|  Note $R_l = R_r = \frac{W_O L}{2}$ and $M_l = M_r = \frac{W_O L^2}{12}$ | $\delta(x) = \frac{W_O x^2}{24EI}(L^2 - 2Lx + x^2)$ | $\lvert\delta_{max}\rvert = \frac{W_O L^4}{384EI}$ at $x = \frac{L}{2}$ | $\lvert\phi_{max}\rvert = 0.008\frac{W_O L^3}{24EI}$ at $x = \frac{L}{2} \pm \frac{L}{\sqrt{12}}$ |

Crandall, S.H. & N.C. Dahl, *An Introduction to The Mechanics of Solids*, McGraw-Hill Book Co., Inc., 1959.

# 10.6 Columns

Long slender members loaded axially in compression are referred to as *columns*. Such members frequently fail by *buckling* (excessive lateral deflection) rather than by crushing. Buckling onset depends not only on the material properties but also the geometry and type of end supports of the column. The axial load at the onset of buckling is called the *critical load*.

If the *slenderness ratio* of the column, defined as $L/r$ (length divided by least radius of gyration $r$ where $r = \sqrt{I/A}$ ), is greater than 120 (it can be as low as 60) the critical load for a column is the Euler load

$$P_{cr} = \pi^2 EI/k^2L^2 \qquad (10.6.1)$$

Values of $k$, with end supports shown in parentheses, are given as

$$\begin{aligned} k &= 1 \quad \text{(pinned - pinned)} \\ k &= 0.5 \quad \text{(fixed - fixed)} \\ k &= 0.7 \quad \text{(pinned - fixed)} \\ k &= 2 \quad \text{(free - fixed)} \end{aligned} \qquad (10.6.2)$$

Intermediate columns are those whose slenderness ratios are less than 120 but greater than that at which failure occurs by crushing. For these, empirical formulas have been developed to predict buckling.

**Example 10.12**

**Determine the critical load for a square steel ($E$ = 210 GPa) strut 8 cm × 8 cm if its length is 6 m under (a) pinned ends, (b) fixed ends.**

**Solution.** The moment of inertia is

$$I = bh^3/12 = (0.08)(0.08)^3/12 = 3.4 \times 10^{-6}\ \text{m}^4$$

a) The critical load for pinned ends is

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(210 \times 10^9)(3.4 \times 10^{-6})}{6^2} = 195\ 000\ \text{N}$$

The normal stress, which must not exceed the yield stress, is

$$\sigma = \frac{F}{A} = \frac{195 \times 10^3}{0.0064} = 30.5 \times 10^6\ \text{Pa}$$

This is substantially less than the yield stress for all steels.

b) The critical load for fixed ends is

$$P_{cr} = \frac{\pi^2(210 \times 10^9)(3.4 \times 10^{-6})}{0.5^2 \times 6^2} = 780\ 000\ \text{N}$$

The normal stress for this case is

$$\sigma = \frac{780 \times 10^3}{0.0064} = 122 \times 10^6\ \text{Pa}$$



# Practice Problems

*(If you choose to work only a few problems, select those with a star.)*

## Stress and Strain

*10.1 A structural member with the same material properties in all directions at any particular point is
a) homogeneous
b) isotropic
c) isentropic
d) holomorphic

*10.2 The amount of lateral strain in a tension member can be calculated using
a) the bulk modulus
b) Poisson's ratio
c) the yield stress
d) Hooke's law

10.3 Wood has grain resulting in material properties quite different normal to the grain compared with properties parallel to the grain. Such a material is
a) nonhomogeneous
b) nonholomorphic
c) nonorthotropic
d) nonisotropic

*10.4 Find the allowable load, in kN, on a 2-cm-dia. 1-m-long, steel rod if its maximum elongation cannot exceed 0.1 cm.
a) 35 b) 45 c) 55 d) 66

10.5 An elevator is suspended by a 2-cm-dia., 30-m-long steel cable. Twenty people, with a total weight of 14 000 N, enter. How far, in millimeters, does the elevator drop?
a) 3.5 b) 4.5 c) 5.5 d) 6.4

10.6 A hole, one meter from the end of a structural steel member fixed at one end, is 0.8 mm shy of matching another hole for possible connection. What force, in kN, is necessary to stretch it for connection? The cross section is 25 mm × 3 mm.
a) 12.6 b) 13.6 c) 14.7 d) 15.8

10.7 As the load is applied, edge $AB$ moves 0.03 mm to the right. Determine the shear modulus, in MPa.
a) 50 300 c) 38 600
b) 41 700 d) 32 500