Review of Engineering Thermodynamics

Universal Balance Equation for Any Extensive Property

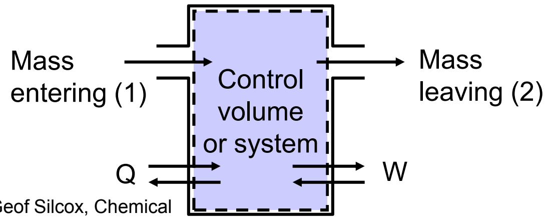
Accumulation = transport + generation

Integrated form for some period of time:

$$\begin{bmatrix}\text{final}\\\text{amount}\end{bmatrix}-\begin{bmatrix}\text{initial}\\\text{amount}\end{bmatrix}=\begin{bmatrix}\text{amount}\\\text{entering}\end{bmatrix}-\begin{bmatrix}\text{amount}\\\text{leaving}\end{bmatrix}+\begin{bmatrix}\text{amount}\\\text{generated}\end{bmatrix}-\begin{bmatrix}\text{amount}\\\text{consumed}\end{bmatrix}$$

Rate form:

$$\begin{bmatrix}\text{rate of}\\\text{change}\end{bmatrix}=\begin{bmatrix}\text{rate of}\\\text{transport in}\end{bmatrix}-\begin{bmatrix}\text{rate of}\\\text{transport out}\end{bmatrix}+\begin{bmatrix}\text{rate of}\\\text{generation}\end{bmatrix}-\begin{bmatrix}\text{rate of}\\\text{consumption}\end{bmatrix}$$



Mass Balance

- Unsteady balance for CV

$$\frac{dm_{CV}}{dt}=\sum_{inlets}\dot{m}_i-\sum_{exits}\dot{m}_e \qquad \Delta m_{CV}=m_2-m_1=\sum_{inlets}m_i-\sum_{exits}m_e$$

- Steady balance for CV

$$0=\sum_{inlets}\dot{m}_i-\sum_{exits}\dot{m}_e \qquad 0=\sum_{inlets}m_i-\sum_{exits}m_e$$

- Balance for closed system

$$\frac{dm_{sys}}{dt}=0 \qquad \Delta m_{sys}=m_2-m_1=0$$

- Averaged flow

$$\dot{m}=\rho_{av}Vel_{av}A=\frac{Vel_{av}A}{v_{av}}=\frac{\dot{V}}{v_{av}}$$

Energy Balance

$$e = u + \frac{1}{2}Vel^2 + gz,\ \ h = u + Pv$$

- Unsteady balance for CV

$$\frac{dE_{CV}}{dt} = \dot{Q}_{in,net} + \dot{W}_{in,net} + \sum_{inlets} \dot{m}_i \left(h + \frac{Vel^2}{2} + gz \right)_i - \sum_{exits} \dot{m}_e \left(h + \frac{Vel^2}{2} + gz \right)_e$$

- Steady balance for CV

$$0 = \dot{Q}_{in,net} + \dot{W}_{in,net} + \sum_{inlets} \dot{m}_i \left(h + \frac{Vel^2}{2} + gz \right)_i - \sum_{exits} \dot{m}_e \left(h + \frac{Vel^2}{2} + gz \right)_e$$

- Balance for closed system

$$\frac{dE_{sys}}{dt} = \dot{Q}_{in,net} + \dot{W}_{in,net}$$

$$\Delta E_{sys} = Q_{in,net} + W_{in,net}$$

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Entropy Balance

There is only one form of entropy.

- Unsteady balance for CV

$$\frac{dS_{CV}}{dt} = \sum_{j=1}^{n} \frac{\dot{Q}_{in,j}}{T_j} + \sum_{inlets} \dot{m}_i s_i - \sum_{exits} \dot{m}_e s_e + \dot{S}_{gen}$$

$\dot{S}_{gen} > 0$ irreversible process

$\dot{S}_{gen} = 0$ reversible process

$\dot{S}_{gen} < 0$ impossible process

- Steady balance for CV

$$0 = \sum_{j=1}^{n} \frac{\dot{Q}_{in,j}}{T_j} + \sum_{inlets} \dot{m}_i s_i - \sum_{exits} \dot{m}_e s_e + \dot{S}_{gen}$$

- Balance for closed system

$$\frac{dS_{sys}}{dt} = \sum_{j=1}^{n} \frac{\dot{Q}_{in,j}}{T_j} + \dot{S}_{gen}$$

$$\Delta S_{sys} = m(s_2 - s_1) = \int_1^2 \frac{\delta Q_{in}}{T} + S_{gen}$$

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Combined Entropy and Energy Balance

$$du = \delta q + \delta w \qquad ds = \frac{\delta q}{T} + ds_{gen}$$

$$du = \delta q_{rev} + \delta w_{rev} \qquad ds = \frac{\delta q_{rev}}{T} \qquad \delta w_{rev} = -Pdv$$

$$Tds = du + Pdv \quad \text{or} \quad ds = \frac{du}{T} + \frac{Pdv}{T}$$

An alternate form follows from the relation

$$d(Pv) = Pdv + vdP$$

$$Tds = dh - vdP \quad \text{or} \quad ds = \frac{dh}{T} - \frac{vdP}{T}$$

Combined Entropy and Energy Balance for Ideal Gases

$$\Delta s = \int_1^2 c_v(T)\frac{dT}{T} + R\ln\frac{v_2}{v_1} \qquad \Delta s = \int_1^2 c_p(T)\frac{dT}{T} - R\ln\frac{P_2}{P_1}$$

For constant or averaged heat capacities,

$$\Delta s = c_{v,av}\ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1} \qquad \Delta s = c_{p,av}\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

where $T_{av} = \frac{T_1 + T_2}{2}$ and $c_{av} = c(T_{av})$

Equations for Work

Reversible boundary work, closed system:

$$w_{rev,in} = -\int_{v_1}^{v_2} P dv$$

Steady-flow, reversible work, open system:

$$w_{rev,in} = \frac{\dot{W}}{\dot{m}} = \int_{P_1}^{P_2} v dP + \frac{Vel_2^2 - Vel_1^2}{2} + g(z_2 - z_1)$$

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Models of Working Substances

Solid/ Liquid

$C_p = C_v = C$

$\Delta u = \Delta h = C\Delta T$

Gases

Phase-change fluids (water & refrigerants)

$$x = \frac{m_{vap}}{m_{tot}} = \frac{v - v_f}{v_g - v_f}$$

Mixtures (advanced)

Perfect gas

$Pv = RT$

P and T are absolute.

$C_p = constant$

$C_v = C_p - R$

$\Delta u = C_v \Delta T$

$\Delta h = C_p \Delta T$.

Ideal gas

$Pv = RT$

P and T are absolute.

$C_p = f(T)$.

$C_v = C_p - R$

$$\Delta h = \int_{T_R}^{T} C_p dT \quad \Delta u = \int_{T_R}^{T} C_v dT$$

Real gas

$$Z = f(T_R, P_R) = \frac{v_{actual}}{v_{ideal}}$$

$$Z = \frac{P v_{actual}}{RT}$$

$$T_R = \frac{T}{T_c} \quad P_R = \frac{P}{P_c}$$

P and T are absolute.

This chart was inspired by Subrata Bhattacharjee, *The Expert System for Thermodynamics*, Prentice Hall, Upper Saddle River, NJ, 2003. ISBN 0-13-009235-5. www.thermofluids.net.

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