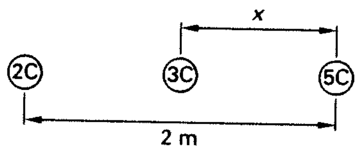
Electrical (9% of morning session questions, 11/120)
= FE - type questions

4 hrs / 120 questions = 2 min each. (ave)

1. The net force on the center charge is zero for the system of three colinear charges shown. What is the distance x?



(A) 0.77 m
(B) 1.11 m
(C) 1.17 m
(D) 1.23 m

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*6.33 The electric flux passing out through a closed surface is equal to

a) the line integral of the current around the surface.
b) zero.
c) the flux density at the surface.
d) the total charge enclosed by the surface.

6.36 A point charge of 2×10^{-7} C is located at the origin of coordinates. A spherical shell with center at the origin and radius of 20 cm has a surface charge density 1×10^{-7} $\mathrm{C/m^2}$. The electric flux density at r = 50 cm, in $\mathrm{C/m^2}$, is

a) 3.18×10^{-8} c) 9.55×10^{-8}
b) 7.96×10^{-8} d) 11.14×10^{-8}

5. A current of 10 A flows through a 1 mm diameter wire. What is the average number of electrons that pass through a cross section of the wire per second?

(A) 1.6×10^{18} e/s
(B) 6.2×10^{18} e/s
(C) 1.6×10^{19} e/s
(D) 6.3×10^{19} e/s

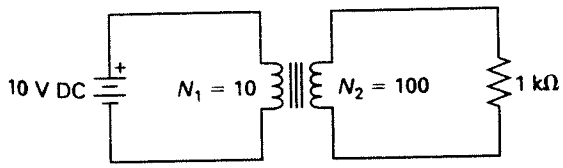
*6.34 The direction of the force acting on a moving charge placed in a magnetic field is

a) perpendicular to the magnetic field.
b) opposite to the direction of motion of the charge.
c) along the direction of the magnetic field.
d) along the direction of motion of the charge.

*6.41 Two long, straight conductors located at (0,3,z) and (0,–3,z) each carry 5 amperes in the same direction (distances are in meters). The magnitude of magnetic field intensity at (4,0,0) is

a) $1/\pi$ b) $2/5\pi$ c) $3/5\pi$ d) $4/5\pi$

14. How much power is dissipated by the 1 kΩ resistor?



(A) 0 W
(B) 0.1 W
(C) 1 W
(D) 10 W

2. A heating element consists of two wires of different materials connected in series. At 20°C, they have resistances of 600 Ω and 300 Ω, and average temperature coefficients of 0.001 1/°C and 0.004 1/°C, respectively. What is the heating element's total resistance at 50°C?

(A) 900 Ω
(B) 950 Ω
(C) 980 Ω
(D) 990 Ω

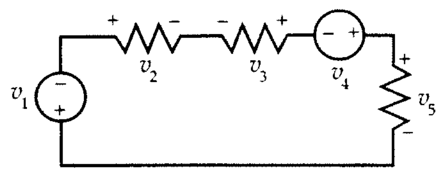
*6.1 For the circuit below, with voltages' polarities as shown, KVL in equation form is

a) $v_1 + v_2 + v_3 - v_4 + v_5 = 0$

b) $-v_1 + v_2 + v_3 - v_4 + v_5 = 0$

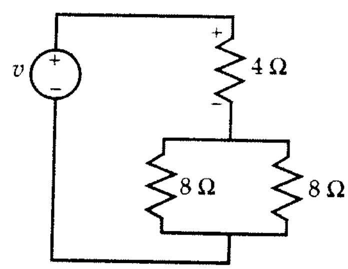
c) $v_1 + v_2 - v_3 - v_4 + v_5 = 0$

d) $-v_1 - v_2 - v_3 + v_4 + v_5 = 0$

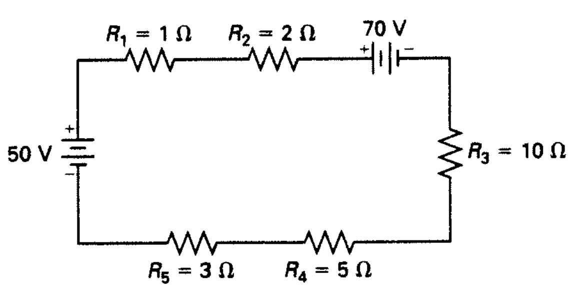


*6.4 For the circuit shown, the voltage across the 4 ohm resistor is, with $v = 1$ V

a) 1/4
b) 1/2
c) 2/3
d) 2



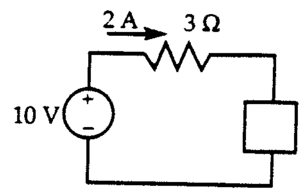
6. What is the voltage across the 10 Ω resistor in the circuit shown?



(A) 9.5 V
(B) 24 V
(C) 33 V
(D) 57 V

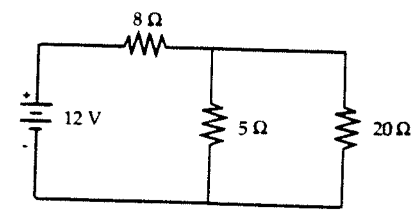
*6.3 Find the magnitude and sign of the power, in watts, absorbed by the circuit element in the box.

a) –20
b) –8
c) 8
d) 12

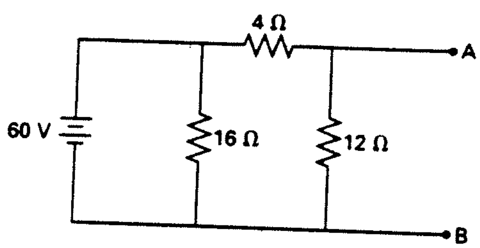


6.7 The power delivered to the 5 ohm resistor is

a) 1.5
b) 2.15
c) 2.85
d) 3.2



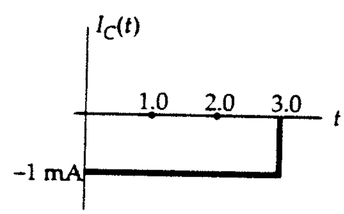
7. What are the Thevenin equivalent resistance and voltage between terminals A and B?



(A) $R_{Th} = 3\ \Omega,\ V_{Th} = 45$ V
(B) $R_{Th} = 7.5\ \Omega,\ V_{Th} = 7.5$ V
(C) $R_{Th} = 7.5\ \Omega,\ V_{Th} = 60$ V
(D) $R_{Th} = 12\ \Omega,\ V_{Th} = 5$ V

6.29 A 100 μF capacitor has $I_C(t)$. The capacitor voltage $V_c(t)$ at $t = 2.5$ seconds $(V(0) = 1.0\text{ V})$ is most nearly

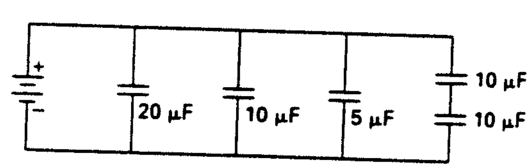
a) –24
b) –25
c) 25
d) 26



6.30 The voltage across a 10 μF capacitor is $50t^2$ V. The time, in seconds, it will take to store 200 J of energy is most nearly

a) 0.15 b) 0.21 c) 1.38 d) 11.25

22. What is the equivalent capacitance seen by the battery for the circuit shown?

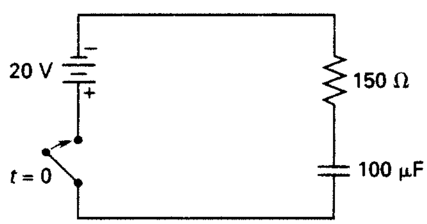


(A) 3 μF
(B) 30 μF
(C) 40 μF
(D) 50 μF

15. A 10-microfarad capacitor has been charged to a potential of 150 volts. A resistor of 25 Ω is then connected across the capacitor through a switch. When the switch is closed for 10 time constants, the total energy dissipated by the resistor is most nearly

(A) 1.0×10^{-7} joules
(B) 1.1×10^{-1} joules
(C) 9.0×10^{1} joules
(D) 9.0×10^{3} joules

28. The switch in the circuit shown is closed at $t = 0$. How long will it take to charge the capacitor to 80% of the battery voltage?



(A) 2.0 ms
(B) 10 ms
(C) 12 ms
(D) 24 ms

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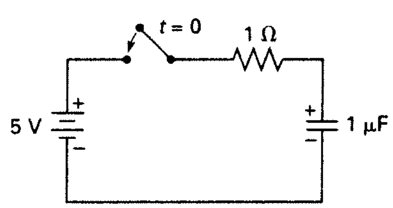
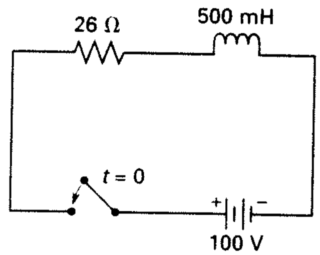
29. The initial voltage across the capacitor is 2.5 V. The switch is closed at $t = 0$. What is the current at $t = 0$ s?



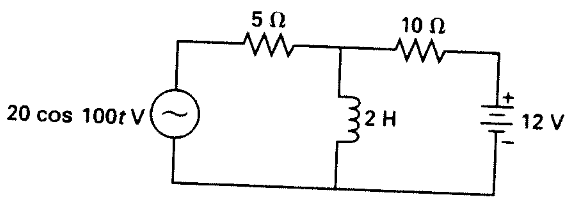
(A) 0.2 A
(B) 0.7 A
(C) 1.0 A
(D) 2.5 A

27. The switch in the circuit shown is closed at $t = 0$. What is the voltage across the inductor at $t = 30$ ms?



(A) 1.0 V
(B) 19 V
(C) 21 V
(D) 48 V

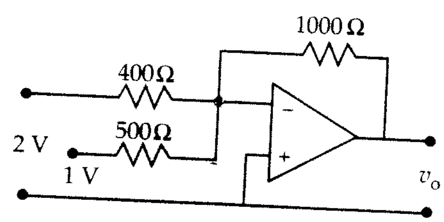
31. **What** is the average DC current through the inductor?



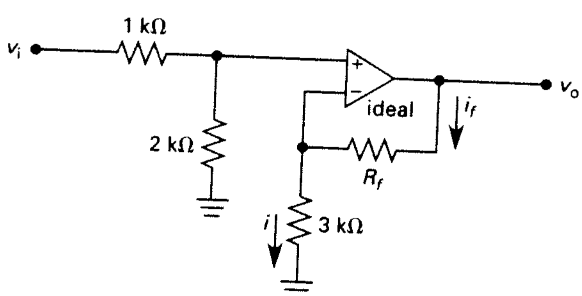
(A) 0 A
(B) 0.8 A
(C) 1.2 A
(D) 3.2 A

6.51 Given the voltages into the following OP-AMP network, the output voltage is

a) –2
b) –4
c) –7
d) –10



4. For the ideal op amp shown, what should be the value of resistor R_f to obtain a gain of 5?



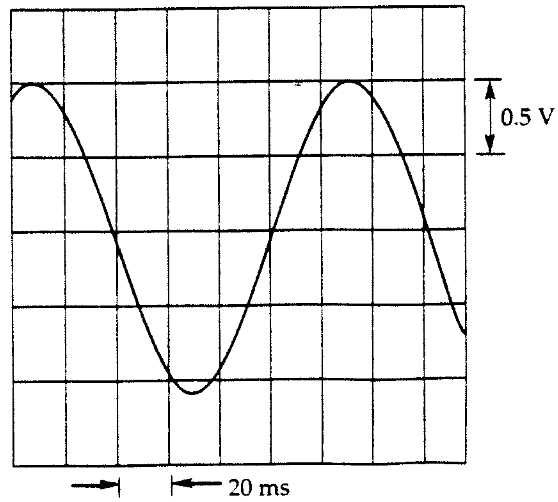
(A) 12.0 kΩ
(B) 19.5 kΩ
(C) 22.5 kΩ
(D) 27.0 kΩ

*6.12 $(2+j2)(3-j4)$ is most nearly

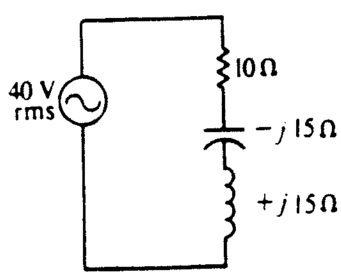
a) $6.0\angle{-21.8^\circ}$ b) $14.1\angle{-21.8^\circ}$ c) $14.1\angle{-8.1^\circ}$ d) $28.0\angle{-8.1^\circ}$

*6.13 The following sinusoid is displayed on an oscilloscope. The RMS voltage and the radian frequency are most nearly

a) 1, 8.33
b) 0.7071, 52.36
c) 1.4142, 52.36
d) 2, 8.33



26.

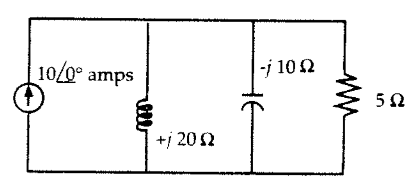


What is the magnitude of the steady-state, root-mean-square voltage across the capacitor in the circuit shown above?

(A) 15 V
(B) 30 V
(C) 45 V
(D) 60 V
(E) 75 V

6.18 The current through the capacitor is

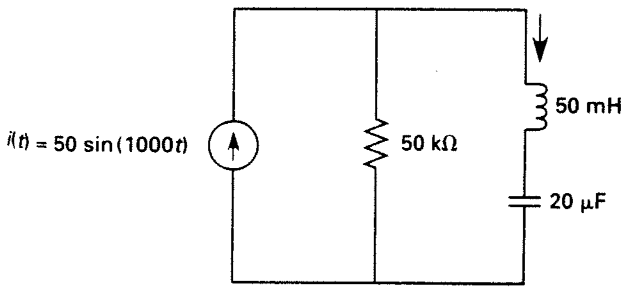
a) 0.21 A
b) 0.57 A
c) 1.0 A
d) 4.85 A



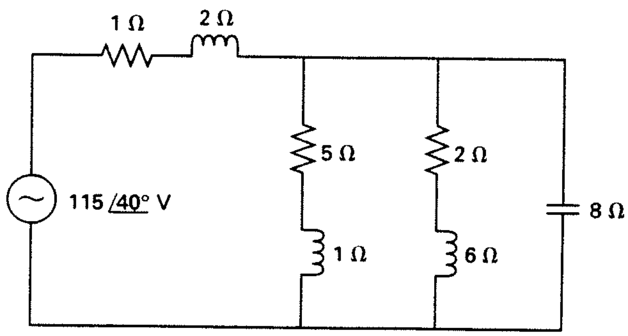
6.19 The voltage across the 5-ohm resistor of Problem 6.18 is

a) 0.50 V
b) 1.61 V
c) 2.06 V
d) 48.5 V

2. What is the current through the LC leg of the following circuit?



(A) 0
(B) $50 \sin(1000t)$ A
(C) $50 \sin\left(1000t + \frac{\pi}{4}\right)$ A
(D) $70.7 \sin\left(1000t - \frac{3\pi}{4}\right)$ A



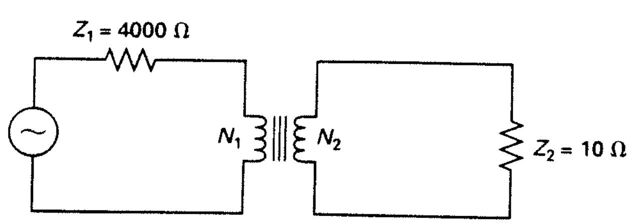
3. What is the average power dissipated by the circuit?

(A) 24 W
(B) 765 W
(C) 910 W
(D) 1970 W

6. A 13.2 kV circuit has a 10 000 kVA load with a 0.85 lagging power factor. How much capacitive reactive power (in kVAR) is needed to correct the power factor to 0.97 lagging?

(A) 2500 kVAR
(B) 3138 kVAR
(C) 4753 kVAR
(D) 5156 kVAR

7. What is the turns ratio (N_1: N_2) for maximum power transfer in the following circuit?



(A) 1:40
(B) 1:20
(C) 20:1
(D) 40:1

Answers

1 D	6.33 d	6.36 b	5 D	6.34 a	6.41 d	14 A	2 B	6.1 c	6.4 b	6 A
6.3 c	6.7 d	7 A	6.29 a	6.30 d	22 c	15 B	28 D	29 D	27 C	31 C
6.51 c	4 B	6.12 c	6.13 b	26 D	6.18 d	6.19 d	2 B	3 d	6 B	7 C

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ELECTRICAL AND COMPUTER ENGINEERING

UNITS

The basic electrical units are coulombs for charge, volts for voltage, amperes for current, and ohms for resistance and impedance.

ELECTROSTATICS

$$\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon r^2}\mathbf{a}_{r12}, \text{ where}$$

$\mathbf{F}_2$ = the force on charge 2 due to charge 1,

Q_i = the ith point charge,

r = the distance between charges 1 and 2,

$\mathbf{a}_{r12}$ = a unit vector directed from 1 to 2, and

ε = the permittivity of the medium.

For free space or air:

$\varepsilon = \varepsilon_o = 8.85 \times 10^{-12}$ farads/meter

Electrostatic Fields

Electric field intensity **E** (volts/meter) at point 2 due to a point charge Q_1 at point 1 is

$$\mathbf{E} = \frac{Q_1}{4\pi\varepsilon r^2}\mathbf{a}_{r12}$$

For a line charge of density ρ_L coulomb/meter on the z-axis, the radial electric field is

$$\mathbf{E}_L = \frac{\rho_L}{2\pi\varepsilon r}\mathbf{a}_r$$

For a sheet charge of density ρ_s coulomb/meter^2 in the x-y plane:

$$\mathbf{E}_s = \frac{\rho_s}{2\varepsilon}\mathbf{a}_z,\ z > 0$$

Gauss' law states that the integral of the electric flux density $\mathbf{D} = \varepsilon\mathbf{E}$ over a closed surface is equal to the charge enclosed or

$$Q_{encl} = \oiint_s \varepsilon\mathbf{E}\cdot d\mathbf{S}$$

The force on a point charge Q in an electric field with intensity **E** is $\mathbf{F} = Q\mathbf{E}$.

The work done by an external agent in moving a charge Q in an electric field from point p_1 to point p_2 is

$$W = -Q\int_{p_1}^{p_2}\mathbf{E}\cdot d\mathbf{l}$$

The energy stored W_E in an electric field **E** is

$$W_E = (1/2)\iiint_V \varepsilon\,|\mathbf{E}|^2\,dV$$

Voltage

The potential difference V between two points is the work per unit charge required to move the charge between the points.

For two parallel plates with potential difference V, separated by distance d, the strength of the E field between the plates is

$$E = \frac{V}{d}$$

directed from the + plate to the – plate.

Current

Electric current $i(t)$ through a surface is defined as the rate of charge transport through that surface or

$i(t) = dq(t)/dt$, which is a function of time t

since $q(t)$ denotes instantaneous charge.

A constant current $i(t)$ is written as I, and the vector current density in amperes/m^2 is defined as **J**.

Magnetic Fields

For a current carrying wire on the z-axis

$$\mathbf{H} = \frac{\mathbf{B}}{\mu} = \frac{I\mathbf{a}_\phi}{2\pi r}, \text{ where}$$

H = the magnetic field strength (amperes/meter),

B = the magnetic flux density (tesla),

$\mathbf{a}_\phi$ = the unit vector in positive ϕ direction in cylindrical coordinates,

I = the current, and

μ = the permeability of the medium.

For air: $\mu = \mu_o = 4\pi \times 10^{-7}$ H/m

Force on a current carrying conductor in a uniform magnetic field is

$$\mathbf{F} = I\mathbf{L} \times \mathbf{B}, \text{ where}$$

L = the length vector of a conductor.

The energy stored W_H in a magnetic field **H** is

$$W_H = (1/2)\iiint_V \mu\,|\mathbf{H}|^2\,dv$$

Induced Voltage

Faraday's Law states for a coil of N turns enclosing flux ϕ:

$v = -N\,d\phi/dt$, where

v = the induced voltage, and

ϕ = the flux (webers) enclosed by the N conductor turns, and

$\phi = \int_S \mathbf{B}\cdot d\mathbf{S}$

Resistivity

For a conductor of length L, electrical resistivity ρ, and cross-sectional area A, the resistance is

$$R = \frac{\rho L}{A}$$

For metallic conductors, the resistivity and resistance vary linearly with changes in temperature according to the following relationships:

$\rho = \rho_o[1 + \alpha(T - T_o)]$, and

$R = R_o[1 + \alpha(T - T_o)]$, where

ρ_o is resistivity at T_o, R_o is the resistance at T_o, and

α is the temperature coefficient.

Ohm's Law: $V = IR$; $v(t) = i(t)\,R$

Resistors in Series and Parallel

For series connections, the current in all resistors is the same and the equivalent resistance for n resistors in series is

$$R_S = R_1 + R_2 + \ldots + R_n$$

For parallel connections of resistors, the voltage drop across each resistor is the same and the equivalent resistance for n resistors in parallel is

$$R_P = 1/(1/R_1 + 1/R_2 + \ldots + 1/R_n)$$

For two resistors R_1 and R_2 in parallel

$$R_P = \frac{R_1 R_2}{R_1 + R_2}$$

Power Absorbed by a Resistive Element

$$P = VI = \frac{V^2}{R} = I^2 R$$

Kirchhoff's Laws

Kirchhoff's voltage law for a closed path is expressed by

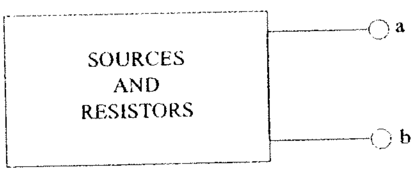
$$\Sigma V_{\text{rises}} = \Sigma V_{\text{drops}}$$

Kirchhoff's current law for a closed surface is

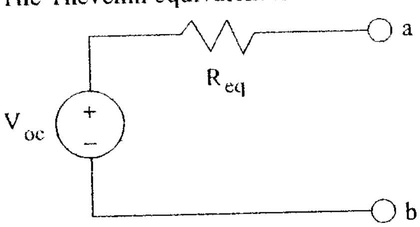
$$\Sigma I_{\text{in}} = \Sigma I_{\text{out}}$$

SOURCE EQUIVALENTS

For an arbitrary circuit



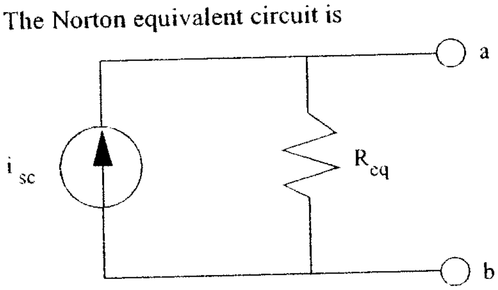
The Thévenin equivalent is



$$R_{eq} = \frac{V_{oc}}{i_{sc}}$$

The open circuit voltage V_{oc} is $V_a - V_b$, and the short circuit current is i_{sc} from a to b.

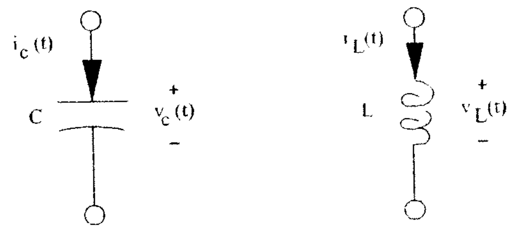
The Norton equivalent circuit is



where i_{sc} and R_{eq} are defined above.

A load resistor R_L connected across terminals a and b will draw maximum power when $R_L = R_{eq}$.

CAPACITORS AND INDUCTORS



The charge $q_C(t)$ and voltage $v_C(t)$ relationship for a capacitor C in farads is

$$C = q_C(t)/v_C(t) \qquad \text{or} \qquad q_C(t) = Cv_C(t)$$

A parallel plate capacitor of area A with plates separated a distance d by an insulator with a permittivity ε has a capacitance

$$C = \frac{\varepsilon A}{d}$$

The current-voltage relationships for a capacitor are

$$v_C(t) = v_C(0) + \frac{1}{C}\int_0^t i_C(\tau)d\tau$$

and $\quad i_C(t) = C\,(dv_C/dt)$

The energy stored in a capacitor is expressed in joules and given by

$$\text{Energy} = Cv_C^2/2 = q_C^2/2C = q_C v_C/2$$

The inductance L of a coil with N turns is

$$L = N\phi/i_L$$

and using Faraday's law, the voltage-current relations for an inductor are

$$v_L(t) = L\,(di_L/dt)$$

$$i_L(t) = i_L(0) + \frac{1}{L}\int_0^t v_L(\tau)d\tau \text{, where}$$

v_L = inductor voltage,
L = inductance (henrys), and
i = inductor current (amperes).

The energy stored in an inductor is expressed in joules and given by

$$\text{Energy} = Li_L^2/2$$

Capacitors and Inductors in Parallel and Series

Capacitors in Parallel

$$C_P = C_1 + C_2 + \ldots + C_n$$

Capacitors in Series

$$C_S = \frac{1}{1/C_1 + 1/C_2 + \ldots + 1/C_n}$$

Inductors in Parallel

$$L_P = \frac{1}{1/L_1 + 1/L_2 + \ldots + 1/L_n}$$

Inductors in Series

$$L_S = L_1 + L_2 + \ldots + L_n$$

AC CIRCUITS

For a sinusoidal voltage or current of frequency f (Hz) and period T (seconds),

$f = 1/T = \omega/(2\pi)$, where

ω = the angular frequency in radians/s.

Average Value

For a periodic waveform (either voltage or current) with period T,

$$X_{ave} = (1/T)\int_0^T x(t)dt$$

The average value of a full-wave rectified sinusoid is

$X_{ave} = (2X_{max})/\pi$

and half this for half-wave rectification, where

X_{max} = the peak amplitude of the waveform.

Effective or RMS Values

For a periodic waveform with period T, the rms or effective value is

$$X_{eff} = X_{rms} = \left[(1/T)\int_0^T x^2(t)\,dt\right]^{1/2}$$

For a sinusoidal waveform and full-wave rectified sine wave,

$$X_{eff} = X_{rms} = X_{max}/\sqrt{2}$$

For a half-wave rectified sine wave,

$X_{eff} = X_{rms} = X_{max}/2$

For a periodic signal,

$$X_{rms} = \sqrt{X_{dc}^2 + \sum_{n=1}^{\infty} X_n^2} \text{ where}$$

X_{dc} is the dc component of $x(t)$

X_n is the rms value of the n[th] harmonic

Sine-Cosine Relations

$\cos(\omega t) = \sin(\omega t + \pi/2) = -\sin(\omega t - \pi/2)$

$\sin(\omega t) = \cos(\omega t - \pi/2) = -\cos(\omega t + \pi/2)$

Phasor Transforms of Sinusoids

$P[V_{max}\cos(\omega t + \phi)] = V_{rms}\angle\phi = \boldsymbol{V}$

$P[I_{max}\cos(\omega t + \theta)] = I_{rms}\angle\theta = \boldsymbol{I}$

For a circuit element, the impedance is defined as the ratio of phasor voltage to phasor current.

$$\mathbf{Z} = \boldsymbol{V}/\boldsymbol{I}$$

For a Resistor, $\mathbf{Z}_R = R$

For a Capacitor, $\mathbf{Z}_C = \dfrac{1}{j\omega C} = jX_C$

For an Inductor,

$Z_L = j\omega L = jX_L$, where

X_C and X_L are the capacitive and inductive reactances respectively defined as

$$X_C = -\frac{1}{\omega C} \quad\text{and}\quad X_L = \omega L$$

Impedances in series combine additively while those in parallel combine according to the reciprocal rule just as in the case of resistors.

ALGEBRA OF COMPLEX NUMBERS

Complex numbers may be designated in rectangular form or polar form. In rectangular form, a complex number is written in terms of its real and imaginary components.

$z = a + jb$, where

a = the real component,

b = the imaginary component, and

$j = \sqrt{-1}$

In polar form

$z = c\angle\theta$, where

$c = \sqrt{a^2 + b^2}$,

$\theta = \tan^{-1}(b/a)$,

$a = c\cos\theta$, and

$b = c\sin\theta$.

Complex numbers are added and subtracted in rectangular form. If

$z_1 = a_1 + jb_1 = c_1(\cos\theta_1 + j\sin\theta_1) = c_1\angle\theta_1$ and

$z_2 = a_2 + jb_2 = c_2(\cos\theta_2 + j\sin\theta_2) = c_2\angle\theta_2$, then

$z_1 + z_2 = (a_1 + a_2) + j(b_1 + b_2)$ and

$z_1 - z_2 = (a_1 - a_2) + j(b_1 - b_2)$

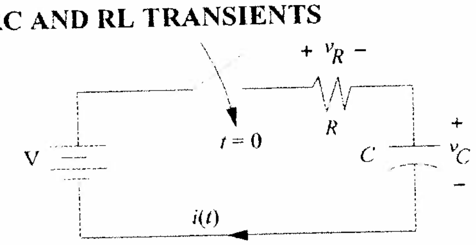
While complex numbers can be multiplied or divided in rectangular form, it is more convenient to perform these operations in polar form.

$z_1 \times z_2 = (c_1 \times c_2)\angle\theta_1 + \theta_2$

$z_1/z_2 = (c_1/c_2)\angle\theta_1 - \theta_2$

The complex conjugate of a complex number $z_1 = (a_1 + jb_1)$ is defined as $z_1^* = (a_1 - jb_1)$. The product of a complex number and its complex conjugate is $z_1z_1^* = a_1^2 + b_1^2$.

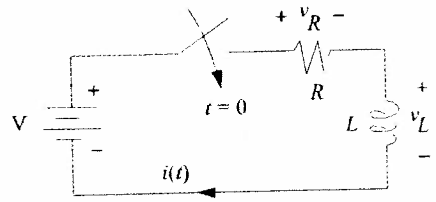
RC AND RL TRANSIENTS



$$t \ge 0;\ v_C(t) = v_C(0)e^{-t/RC} + V(1 - e^{-t/RC})$$

$$i(t) = \{[V - v_C(0)]/R\}e^{-t/RC}$$

$$v_R(t) = i(t)\,R = [V - v_C(0)]e^{-t/RC}$$



$$t \ge 0;\ i(t) = i(0)e^{-Rt/L} + \frac{V}{R}\left(1 - e^{-Rt/L}\right)$$

$$v_R(t) = i(t)\,R = i(0)\,Re^{-Rt/L} + V\,(1 - e^{-Rt/L})$$

$$v_L(t) = L\,(di/dt) = -\,i(0)\,Re^{-Rt/L} + Ve^{-Rt/L}$$

where $v(0)$ and $i(0)$ denote the initial conditions and the parameters RC and L/R are termed the respective circuit time constants.

RESONANCE

The radian resonant frequency for both parallel and series resonance situations is

$$\omega_o = \frac{1}{\sqrt{LC}} = 2\pi f_o \text{ (rad/s)}$$

Series Resonance

$$\omega_o L = \frac{1}{\omega_o C}$$

$Z = R$ at resonance.

$$Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

$BW = \omega_o/Q$ (rad/s)

Parallel Resonance

$$\omega_o L = \frac{1}{\omega_o C} \quad \text{and}$$

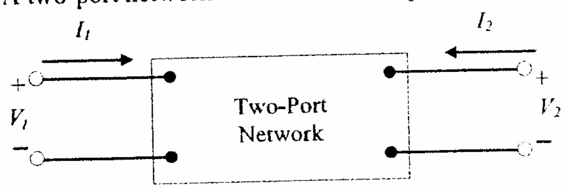
$Z = R$ at resonance.

$$Q = \omega_o RC = \frac{R}{\omega_o L}$$

$BW = \omega_o/Q$ (rad/s)

TWO-PORT PARAMETERS

A two-port network consists of two input and two output terminals as shown below.



A two-port network may be represented by an equivalent circuit using a set of two-port parameters. Three commonly used sets of parameters are impedance, admittance, and hybrid parameters. The following table describes the equations used for each of these sets of parameters.

Parameter Type	Equations	Definitions
Impedance (z)	$V_1 = z_{11}I_1 + z_{12}I_2$ $V_2 = z_{21}I_1 + z_{22}I_2$	$z_{11} = \left.\frac{V_1}{I_1}\right\vert_{I_2=0}$ $z_{12} = \left.\frac{V_1}{I_2}\right\vert_{I_1=0}$ $z_{21} = \left.\frac{V_2}{I_1}\right\vert_{I_2=0}$ $z_{22} = \left.\frac{V_2}{I_2}\right\vert_{I_1=0}$
Admittance (y)	$I_1 = y_{11}V_1 + y_{12}V_2$ $I_2 = y_{21}V_1 + y_{22}V_2$	$y_{11} = \left.\frac{I_1}{V_1}\right\vert_{V_2=0}$ $y_{12} = \left.\frac{I_1}{V_2}\right\vert_{V_1=0}$ $y_{21} = \left.\frac{I_2}{V_1}\right\vert_{V_2=0}$ $y_{22} = \left.\frac{I_2}{V_2}\right\vert_{V_1=0}$
Hybrid (h)	$V_1 = h_{11}I_1 + h_{12}V_2$ $I_2 = h_{21}I_1 + h_{22}V_2$	$h_{11} = \left.\frac{I_1}{V_1}\right\vert_{V_2=0}$ $h_{12} = \left.\frac{V_1}{V_2}\right\vert_{I_1=0}$ $h_{21} = \left.\frac{I_2}{I_1}\right\vert_{V_2=0}$ $h_{22} = \left.\frac{I_2}{V_2}\right\vert_{I_1=0}$

AC POWER

Complex Power

Real power P (watts) is defined by

$$P = (1/2) V_{max} I_{max} \cos\theta$$
$$= V_{rms} I_{rms} \cos\theta$$

where θ is the angle measured from V to I. If I leads (lags) V, then the power factor ($p.f.$),

$$p.f. = \cos\theta$$

is said to be a leading (lagging) $p.f.$

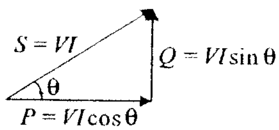
Reactive power Q (vars) is defined by

$$Q = (1/2) V_{max} I_{max} \sin\theta$$
$$= V_{rms} I_{rms} \sin\theta$$

Complex power S (volt-amperes) is defined by

$$S = VI^* = P + jQ,$$

where I^* is the complex conjugate of the phasor current.



Complex Power Triangle (Inductive Load)

For resistors, $\theta = 0$, so the real power is

$$P = V_{rms} I_{rms} = V_{rms}^2 / R = I_{rms}^2 R$$

Balanced Three-Phase (3-ϕ) Systems

The 3-ϕ line-phase relations are

for a delta

$$V_L = V_p$$
$$I_L = \sqrt{3} I_p$$

for a wye

$$V_L = \sqrt{3} V_p = \sqrt{3} V_{LN}$$
$$I_L = I_p$$

where subscripts L/P denote line/phase respectively.

A balanced 3-ϕ delta-connected load impedance can be converted to an equivalent wye-connect load impedance using the following relationship

$$Z_\Delta = 3 Z_Y$$

The following formulas can be used to determine 3-ϕ power for balanced systems.

$$\mathbf{S} = P + jQ$$
$$|\mathbf{S}| = 3 V_P I_P = \sqrt{3} V_L I_L$$
$$\mathbf{S} = 3 \mathbf{V}_P \mathbf{I}_P^* = \sqrt{3} V_L I_L (\cos\theta_P + j \sin\theta_P)$$

For balanced 3-ϕ wye- and delta-connected loads

$$\mathbf{S} = \frac{V_L^2}{Z_Y^*} \qquad \mathbf{S} = 3 \frac{V_L^2}{Z_\Delta^*}$$

where

$\mathbf{S}$ = total 3-ϕ complex power (VA)

$|\mathbf{S}|$ = total 3-ϕ apparent power (VA)

P = total 3-ϕ real power (W)

Q = total 3-ϕ reactive power (var)

θ_P = power factor angle of each phase

V_L = rms value of the line-to-line voltage

V_{LN} = rms value of the line-to-neutral voltage

I_L = rms value of the line current

I_P = rms value of the phase current

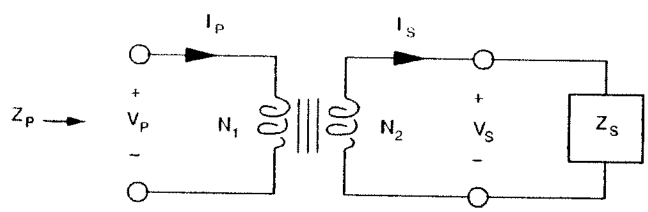
For a 3-ϕ wye-connected source or load with line-to-neutral voltages

$$\mathbf{V}_{an} = V_P \angle 0°$$
$$\mathbf{V}_{bn} = V_P \angle -120°$$
$$\mathbf{V}_{cn} = V_P \angle +120°$$

The corresponding line-to-line voltages are

$$\mathbf{V}_{ab} = \sqrt{3} V_P \angle 30°$$
$$\mathbf{V}_{bc} = \sqrt{3} V_P \angle -90°$$
$$\mathbf{V}_{ca} = \sqrt{3} V_P \angle +150°$$

Transformers (Ideal)



Turns Ratio

$$a = N_1 / N_2$$
$$a = \left|\frac{\mathbf{V}_P}{\mathbf{V}_S}\right| = \left|\frac{\mathbf{I}_S}{\mathbf{I}_P}\right|$$

The impedance seen at the input is

$$\mathbf{Z}_P = a^2 \mathbf{Z}_S$$

AC Machines

The synchronous speed n_s for ac motors is given by

$n_s = 120f/p$, where

f = the line voltage frequency in Hz and

p = the number of poles.

The slip for an induction motor is

slip $= (n_s - n)/n_s$, where

n = the rotational speed (rpm).

DC Machines

The armature circuit of a dc machine is approximated by a series connection of the armature resistance R_a, the armature inductance L_a, and a dependent voltage source of value

$V_a = K_a n\phi$ volts, where

K_a = constant depending on the design,

n = is armature speed in rpm, and

ϕ = the magnetic flux generated by the field.

The field circuit is approximated by the field resistance R_f in series with the field inductance L_f. Neglecting saturation, the magnetic flux generated by the field current I_f is

$\phi = K_f I_f$ webers

The mechanical power generated by the armature is

$P_m = V_a I_a$ watts

where I_a is the armature current. The mechanical torque produced is

$T_m = (60/2\pi)K_a\phi I_a$ newton-meters.

ELECTROMAGNETIC DYNAMIC FIELDS

The integral and point form of Maxwell's equations are

$$\oint \mathbf{E}\cdot d\mathbf{l} = -\iint_S (\partial \mathbf{B}/\partial t)\cdot d\mathbf{S}$$

$$\oint \mathbf{H}\cdot d\mathbf{l} = I_{enc} + \iint_S (\partial \mathbf{D}/\partial t)\cdot d\mathbf{S}$$

$$\oiint_{S_V} \mathbf{D}\cdot d\mathbf{S} = \iiint_V \rho\, dv$$

$$\oiint_{S_V} \mathbf{B}\cdot d\mathbf{S} = 0$$

$$\nabla\times\mathbf{E} = -\partial\mathbf{B}/\partial t$$

$$\nabla\times\mathbf{H} = \mathbf{J} + \partial\mathbf{D}/\partial t$$

$$\nabla\cdot\mathbf{D} = \rho$$

$$\nabla\cdot\mathbf{B} = 0$$

The sinusoidal wave equation in **E** for an isotropic homogeneous medium is given by

$$\nabla^2\mathbf{E} = -\omega^2\mu\varepsilon\mathbf{E}$$

The *EM* energy flow of a volume V enclosed by the surface S_V can be expressed in terms of the Poynting's Theorem

$$-\oiint_{S_V}(\mathbf{E}\times\mathbf{H})\cdot d\mathbf{S} = \iiint_V \mathbf{J}\cdot\mathbf{E}\, dv + \partial/\partial t\{\iiint_V (\varepsilon E^2/2 + \mu H^2/2)\, dv\}$$

where the left-side term represents the energy flow per unit time or power flow into the volume V, whereas the $\mathbf{J}\cdot\mathbf{E}$ represents the loss in V and the last term represents the rate of change of the energy stored in the **E** and **H** fields.

LOSSLESS TRANSMISSION LINES

The wavelength, λ, of a sinusoidal signal is defined as the distance the signal will travel in one period.

$$\lambda = \frac{U}{f}$$

where U is the velocity of propagation and f is the frequency of the sinusoid.

The characteristic impedance, Z_0, of a transmission line is the input impedance of an infinite length of the line and is given by

$$Z_0 = \sqrt{L/C}$$

where L and C are the per unit length inductance and capacitance of the line.

The reflection coefficient at the load is defined as

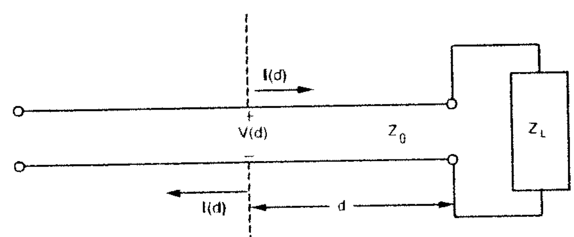
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

and the standing wave ratio SWR is

$$\text{SWR} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

β = Propagation constant $= \dfrac{2\pi}{\lambda}$

For sinusoidal voltages and currents:



Voltage across the transmission line:

$$V(d) = V^+e^{j\beta d} + V^-e^{-j\beta d}$$

Current along the transmission line:

$$I(d) = I^+e^{j\beta d} + I^-e^{-j\beta d}$$

where $I^+ = V^+/Z_0$ and $I^- = -V^-/Z_0$

Input impedance at d

$$Z_{in}(d) = Z_0\frac{Z_L + jZ_0\tan(\beta d)}{Z_0 + jZ_L\tan(\beta d)}$$