

Electrical Review Lecture Fundamentals of Engineering (FE)

Arn Stolp
Exam Information downloaded 4/14
from: www.ncees.org

You can download exam specs, and a reference manual at ncees.org. You can also register for the exam, take a practice test, and pay for it all. The computer-based FE exam costs \$225 and takes 6 hours. This includes a tutorial, 5 hours and 20 minutes to answer 110 questions, and a 25 minute break. That's about 3 minutes per question. All questions are multiple choice and discipline-specific to the student's major. General knowledge topics, such as mathematics, appear on exams for all disciplines. You will take the FE exam on a 24 inch split-screen computer monitor. Half the screen will show the exam questions, and half will display the reference manual. The manual will be a searchable PDF. If you aren't sure of an answer to a question, you can bookmark it and return later, but don't leave it blank in the end. Guessing doesn't hurt you. Because you have to move through the exam quickly, FE questions are designed to be answered quickly. Many of them cover simple conceptual information. Read the problem carefully and ask yourself, "What's the concept they're testing for?"

Search online for information and YouTube videos to review for specific areas of the exam.

Calculator policy (2014) <http://ncees.org/exams/calculator-policy/>

Casio: All fx-115 models. Any Casio calculator must contain fx-115 in its model name.

Examples of acceptable Casio fx-115 models include (but are not limited to):

fx-115 MS fx-115 MS Plus fx-115 MS SR fx-115 ES fx-115 ES Plus

Hewlett Packard: The HP 33s and HP 35s models, but no others.

Texas Instruments: All TI-30X and TI-36X models. Any Texas Instruments calculator must contain either TI-30X or TI-36X in its model name. Examples of acceptable TI-30X and TI-36X models include (but are not limited to):

TI-30Xa TI-30Xa SOLAR TI-30Xa SE TI-30XS Multiview TI-30X IIB TI-30X IIS
TI-36X II TI-36X SOLAR TI-36X Pro

Civil: No electrical questions.

Chemical: Basics (3D), Process control (14)

Other Disciplines: Basics (14), Instrumentation (4)

Mechanical: Basics with motors and generators (6), Instrumentation (14)

Electrical: download "FE-Ele-CBT-specs_with-ranges.pdf" from www.ncees.org

This Lecture will touch on the subjects below:

MECHANICAL CBT Exam Specifications

of quest

6. Electricity and Magnetism

3-5



- A. Charge, current, voltage, power, and energy
- B. Current and voltage laws (Kirchhoff, Ohm)
- C. Equivalent circuits (series, parallel)
- D. AC circuits
- E. Motors and generators

Answers to problems in following pages

1. (D) 2. (A) 3. (B) 4. (A) 5. (D) 6. (D) 7. (A) 8. (C) 9. (B) 10. (D) 11. (C)
12. (B) 13. (A) 14. (C) 15. (A) 16. (C) 17. (D) 18. (C) 19. (D) 20. (A) 21. (B)

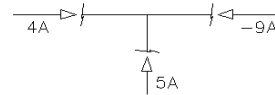
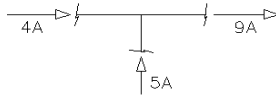
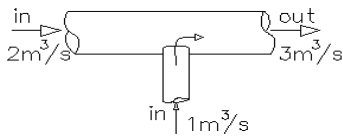
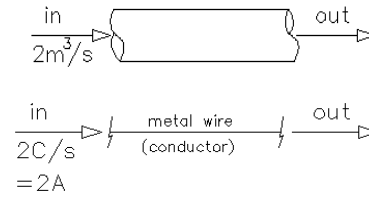
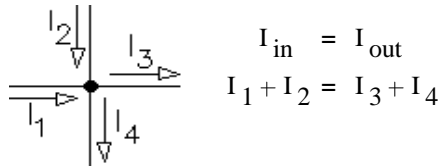
Electrical Engineering FE Review Lecture

A. Stolp
4/24/15

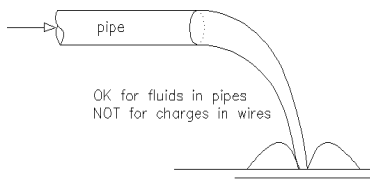
Basic electrical quantities	Letter used	Units	Fluid Analogy
Charge, actually moves	Q	Coulomb (C)	m^3
Current, like fluid flow	$I = \frac{Q}{\text{sec}}$	Amp (A, mA, μA ,...)	$\frac{m^3}{\text{sec}}$
Voltage, like pressure	V or E	volt (V, mV, kV,...)	$\text{Pa} = 1 \cdot \frac{N}{m^2}$
Resistance 	$R = \frac{V}{I}$	Ohm (Ω , k Ω , M Ω ,...)	
Conductance 	$G = \frac{1}{R}$	Siemens (S, also mho, old unit)	
Power = energy/time	$P = V \cdot I$	Watt (W, mW, kW, MW,...)	W

KCL, Kirchhoff's Current Law

$I_{\text{in}} = I_{\text{out}}$ of any point, part, or section

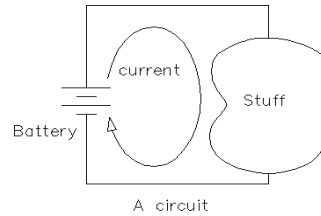


negative current means the direction arrow is wrong

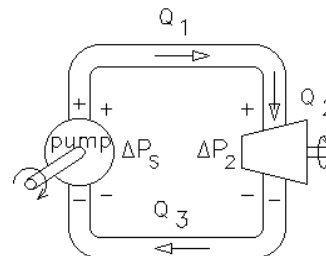
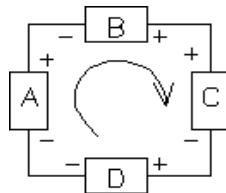
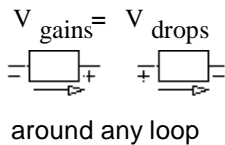


Conductors vs Nonconductors

Battery also obeys KCL. No accumulation of charge anywhere, so it must circulate around. Leads to the concept of a "Circuit"

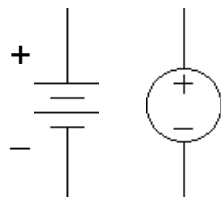


Voltage is like pressure KVL, Kirchhoff's Voltage Law

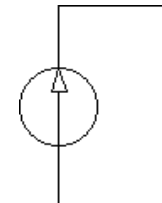


Sources

Ideal batteries or voltage sources always make the same voltage regardless of current.



Ideal current sources always make the same current flow regardless of the voltage



Must have a path for the current to flow

Resistors

Ohm's law

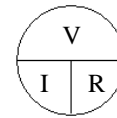


$$I = \frac{V}{R}$$

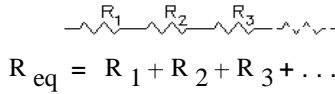
$$R = \frac{V}{I}$$

definition of resistance and the unit " Ω "

Ideal wires have no resistance



series:



$$R_{eq} = R_1 + R_2 + R_3 + \dots$$

Exactly the **same current** through each resistor

Voltage divider:

$$V_{Rn} = V_{total} \cdot \frac{R_n}{R_1 + R_2 + R_3 + \dots}$$

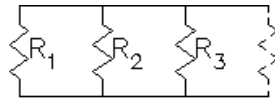
parallel:

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots}$$

Exactly the **same voltage** across each resistor

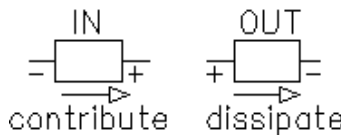
current divider:

$$I_{Rn} = I_{total} \cdot \frac{\frac{1}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots}$$



Power

$P_{IN} = P_{OUT}$ for resistor circuits

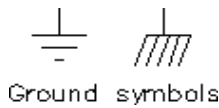


$P = V \cdot I$ for everything

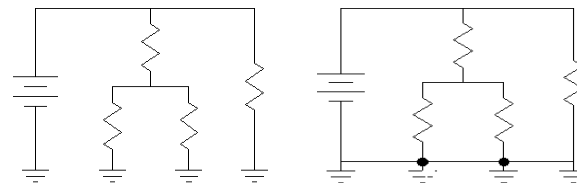
$= I^2 \cdot R = \frac{V^2}{R}$ for resistors

Energy (Joules) = Power X time

Ground

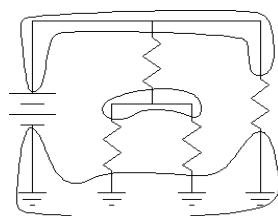


Ground is considered zero volts and is a reference for other voltages.



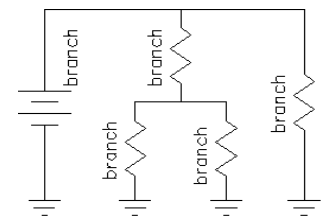
Nodes & Branches

Node = all points connected by wire, all at same voltage (potential)

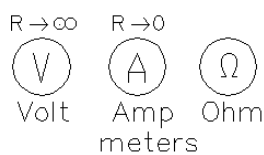


ground is a node

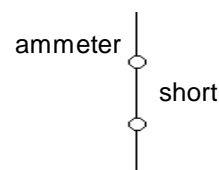
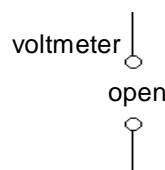
Branch = all parts with the same current



Meters



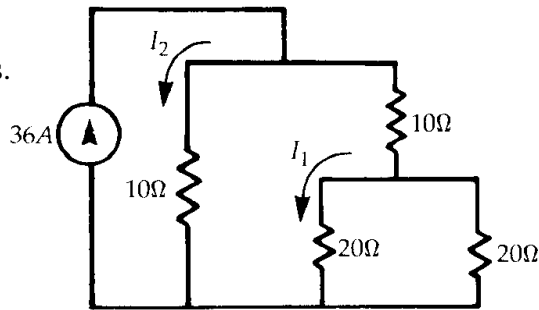
ideally:



Examples

1. Find I_2 in amps.

- (A) 9
- (B) 12
- (C) 18
- (D) 24
- (E) 27

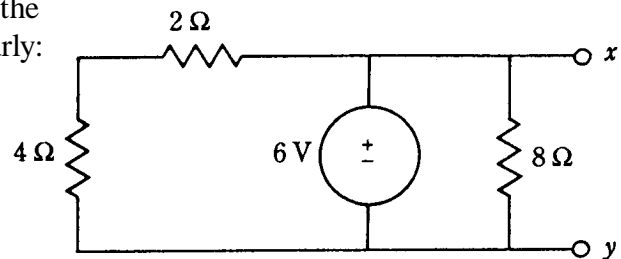


$$36 \cdot A \cdot \frac{\frac{1}{10}}{\frac{1}{10} + \frac{1}{20}} = 24 \cdot A \quad (D)$$

2. If a 12-ohm resistor is connected across terminals xy in the circuit shown, the current through it would be most nearly:

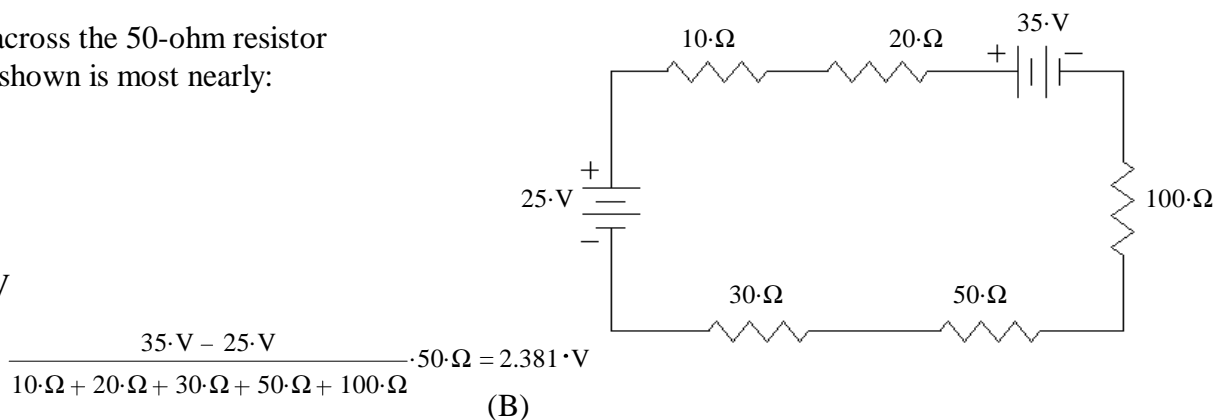
- (A) 0.5 A
- (B) 1.25 A
- (C) 2.0 A
- (D) 2.25 A
- (E) 5.75 A

$$\frac{6 \cdot V}{12 \cdot \Omega} = 0.5 \cdot A \quad (A)$$



3. The voltage across the 50-ohm resistor in the circuit shown is most nearly:

- (A) 1.43 V
- (B) 2.4 V
- (C) 5.95 V
- (D) 8.33 V
- (E) 8.57 V

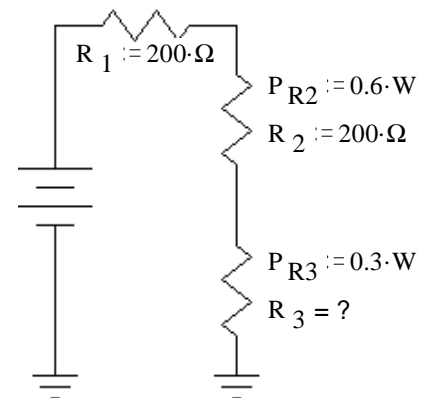


$$\frac{35 \cdot V - 25 \cdot V}{10 \cdot \Omega + 20 \cdot \Omega + 30 \cdot \Omega + 50 \cdot \Omega + 100 \cdot \Omega} \cdot 50 \cdot \Omega = 2.381 \cdot V \quad (B)$$

4. In the circuit shown, the power loss in R_2 is 0.6 W and the power loss in R_3 is 0.3 W. What is the value of the resistor, R_3 ?

- (A) 100 Ω
- (B) 141 Ω
- (C) 283 Ω
- (D) 400 Ω

$P = I^2 \cdot R$
 Power is directly proportional to the resistor
 $R_2 := \frac{0.3 \cdot W}{0.6 \cdot W} \cdot 200 \cdot \Omega \quad R_2 = 100 \cdot \Omega \quad (A)$



5. In the circuit above, what is output power of the battery?

- (A) 0.6 W
- (B) 0.9 W
- (C) 1.2 W
- (D) 1.5 W

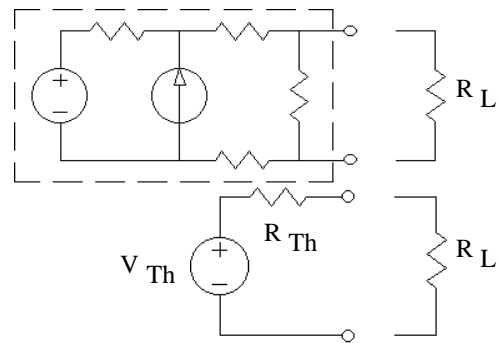
$P_S := 2 \cdot P_{R2} + P_{R3}$
 $P_S = 1.5 \cdot W \quad (D)$

Thévenin Equivalent Circuit

A simplified model can be used for any combination of sources and resistors.

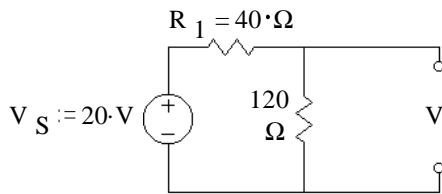
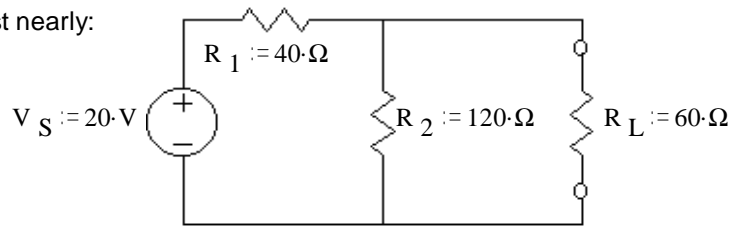
To calculate a circuit's Thévenin equivalent:

- 1) Remove the load and calculate the open-circuit voltage where the load used to be. This is the Thévenin voltage (V_{Th}).
- 2) Zero all the sources. (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
- 3) Compute the total resistance between the load terminals. (DO NOT include the load in this resistance.) This is the Thévenin source resistance (R_{Th}).
- 4) Draw the Thévenin equivalent circuit and add your values.



6. The Thévenin voltage (V_{Th}) of the circuit shown is most nearly:

- (A) 0 V
- (B) 2.4 V
- (C) 5 V
- (D) 15 V
- (E) 20 V

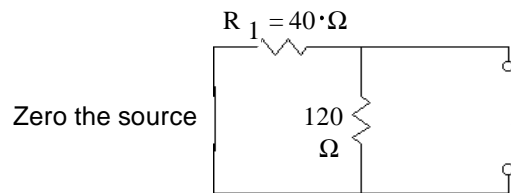


Find the open circuit voltage:

$$V_{oc} = V_{Th} := V_S \cdot \frac{R_2}{R_1 + R_2} \quad V_{Th} = 15 \cdot V \quad (D)$$

7. The Thévenin resistance (R_{Th}) of the circuit above is most nearly:

- (A) 30 Ω
- (B) 40 Ω
- (C) 120 Ω
- (D) 160 Ω



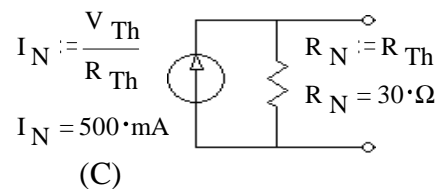
Find the Thevenin resistance:

$$R_{Th} := \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \quad R_{Th} = 30 \cdot \Omega \quad (A)$$

8. The Norton current (I_N) of the circuit above is most nearly:

- (A) 0.125 A
- (B) 0.25 A
- (C) 0.5 A
- (D) 0.67 A

Norton equivalent circuit:



Nodal Analysis Steps

- 1) If the circuit doesn't already have a ground, label one node as ground (zero voltage).
If the ground can be defined as one side of a voltage source, that will make the following steps easier.
Label the remaining node, either with known voltages or with letters, a, b,
- 2) Label unknown node voltages as V_a, V_b, \dots and label the current in each resistor as I_1, I_2, \dots
- 3) Write Kirchoff's current equations for each unknown node.
- 4) Replace the currents in your **KCL** equations with expressions like this. $\frac{V_a - V_b}{R_1}$ Ohm's law relationship using the nodal voltages.
- 5) Solve the multiple equations for the multiple unknown voltages.

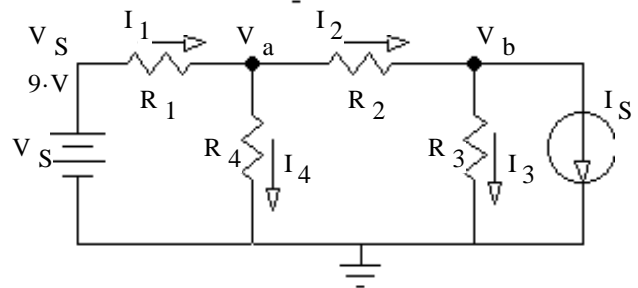
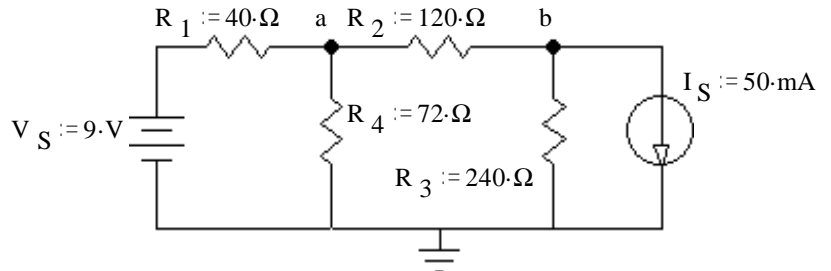
9. The nodal equation for node b is:

(A) $\frac{9 \cdot V - V_a}{40 \cdot \Omega} = \frac{V_a - V_b}{120 \cdot \Omega} + \frac{V_a}{72 \cdot \Omega}$

(B) $\frac{V_a - V_b}{120 \cdot \Omega} = \frac{V_b}{240 \cdot \Omega} + 50 \cdot \text{mA}$

(C) $\frac{V_b - V_a}{120 \cdot \Omega} = \frac{V_b}{240 \cdot \Omega} + 50 \cdot \text{mA}$

(D) $\frac{9 \cdot V - V_a}{40 \cdot \Omega} - \frac{V_a}{72 \cdot \Omega} + \frac{V_a - V_b}{120 \cdot \Omega} = \frac{V_b - 0 \cdot V}{240 \cdot \Omega} + 50 \cdot \text{mA}$



Write Kirchoff's current equations for each unknown node.

node a $I_1 = I_2 + I_4$ unneeded

node b $I_2 = I_3 + I_S$

$\frac{V_a - V_b}{R_2} = \frac{V_b - 0 \cdot V}{R_3} + I_S$ (B)

Sinusoidal AC

T = Period = repeat time

f = frequency, cycles / second $f = \frac{1}{T} = \frac{\omega}{2 \cdot \pi}$

ω = radian frequency, radians/sec $\omega = 2 \cdot \pi \cdot f$

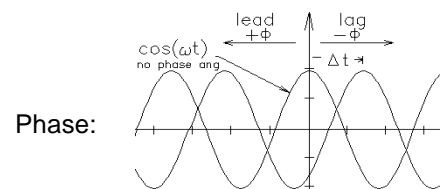
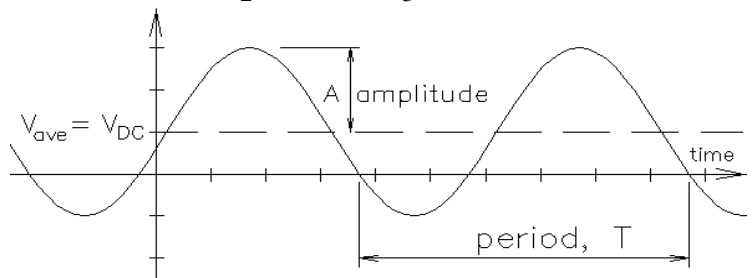
A = amplitude

DC = average RMS: $\frac{A}{\sqrt{2}}$

$y(t) = A \cdot \cos(\omega \cdot t + \phi)$

voltage: $v(t) = V_p \cdot \cos(\omega \cdot t + \phi)$ current: $i(t) = I_p \cdot \cos(\omega \cdot t + \phi)$

Phase: $\phi = -\frac{\Delta t}{T} \cdot 360 \cdot \text{deg}$ or: $\phi = -\frac{\Delta t}{T} \cdot 2 \cdot \pi \cdot \text{rad}$



10. For the following waveforms, find v(t):

(A) $v(t) := 6 \cdot V \cdot \cos(167 \cdot t - 90 \cdot \text{deg}) + 3 \cdot V$

(B) $v(t) := 3 \cdot V \cdot \cos(167 \cdot t - 90 \cdot \text{deg}) + 6 \cdot V$

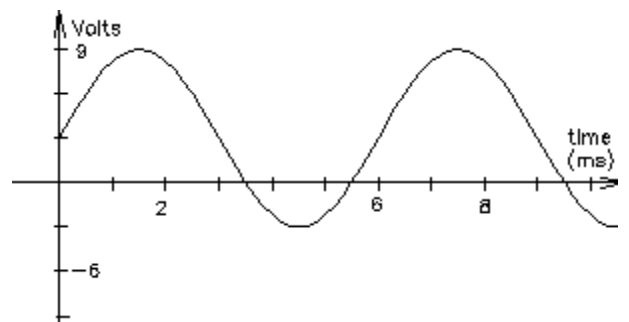
(C) $v(t) := 6 \cdot V \cdot \cos(1047 \cdot t + 90 \cdot \text{deg}) + 3 \cdot V$

(D) $v(t) := 6 \cdot V \cdot \cos(1047 \cdot t - 90 \cdot \text{deg}) + 3 \cdot V$

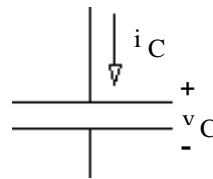
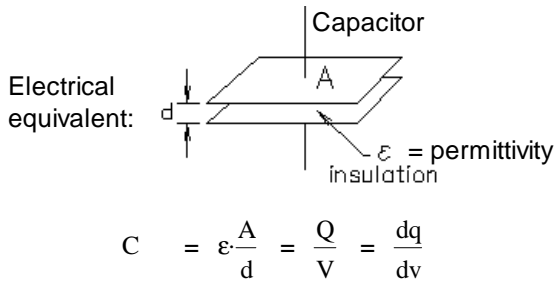
$V_{pp} := 9 \cdot V - -3 \cdot V$ $V_{pp} = 12 \cdot V$ $V_{DC} := 3 \cdot V$

$T := 6 \cdot \text{ms}$ $f := \frac{1}{T}$ $f = 166.7 \cdot \text{Hz}$ $\omega = 2 \cdot \pi \cdot f = 1047 \cdot \frac{\text{rad}}{\text{s}}$

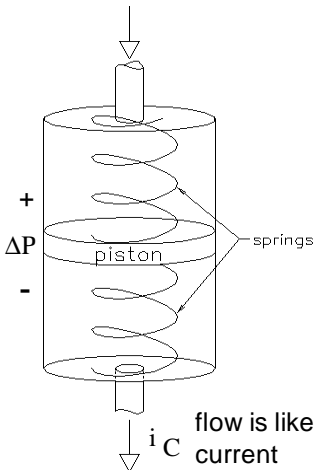
$\phi = -\frac{1.5 \cdot \text{ms}}{6 \cdot \text{ms}} \cdot 360 \cdot \text{deg} = -90 \cdot \text{deg}$ (D)



Capacitors



Fluid Model:



Basic equations you should know:

$$C = \frac{Q}{V} \quad v_C = \frac{1}{C} \int_{-\infty}^t i_C dt = \frac{1}{C} \int_0^t i_C dt + v_C(0) \quad i_C = C \cdot \frac{d}{dt} v_C$$

initial voltage

Units: farad = $\frac{\text{coul}}{\text{volt}} = \frac{\text{amp} \cdot \text{sec}}{\text{volt}}$

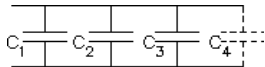
$\mu\text{F} = 1 \cdot 10^{-6} \cdot \text{farad}$

$\text{pF} = 1 \cdot 10^{-12} \cdot \text{farad}$

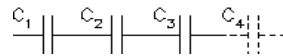
Capacitor voltage **cannot** change instantaneously

Energy stored in electric field: $W_C = \frac{1}{2} \cdot C \cdot V_C^2$

parallel: $C_{eq} = C_1 + C_2 + C_3 + \dots$

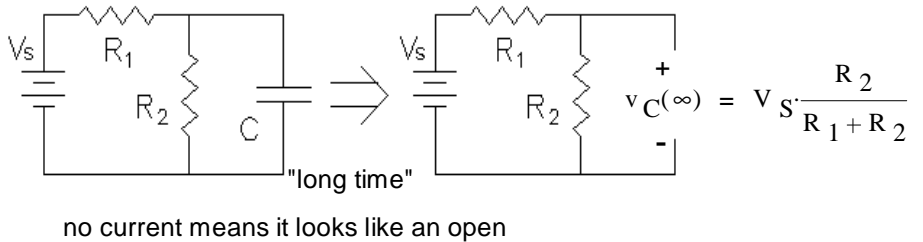


series: $C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots}$



Steady-state or Final conditions

If a circuit has been connected for "a long time", then it has reached a steady state condition. that means the currents and voltages are no longer changing.

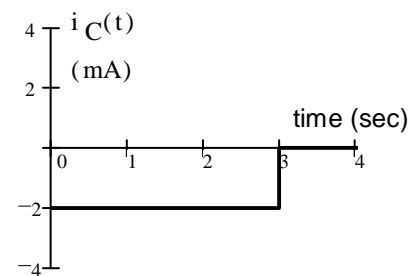


$$\frac{d}{dt} v_C = 0 \quad i_C = C \cdot \frac{d}{dt} v_C = 0$$

11. The current shown flows through a 50- μF capacitor.
The initial voltage on the capacitor is 10V ($v_C(0) = 10\text{V}$).
The capacitor voltage at 2.5 seconds is most nearly:

- (A) -110 V
- (B) -100 V
- (C) -90 V
- (D) 110 V

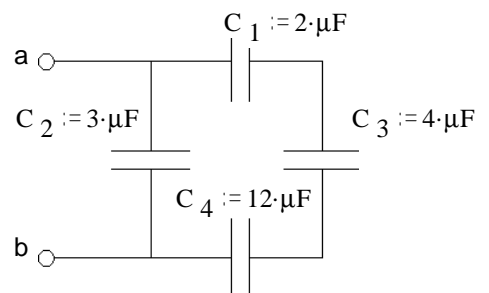
$$\frac{1}{50 \cdot \mu\text{F}} \cdot 2.5 \cdot \text{sec} \cdot (-2 \cdot \text{mA}) + 10 \cdot \text{V} = -90 \cdot \text{V} \quad \text{(C)}$$



12. Find C_{eq} between terminals a and b.

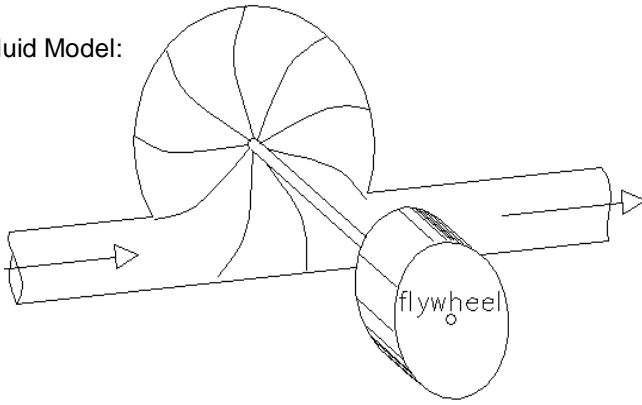
- (A) 3.86 μF
- (B) 4.2 μF
- (C) 4.67 μF
- (D) 21 μF

$$C_{eq} = 3 \cdot \mu\text{F} + \frac{1}{\frac{1}{2 \cdot \mu\text{F}} + \frac{1}{4 \cdot \mu\text{F}} + \frac{1}{12 \cdot \mu\text{F}}} = 4.2 \cdot \mu\text{F} \quad \text{(B)}$$

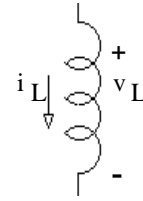


Inductors

Fluid Model:



Electrical equivalent:



$$L = \mu_0 \cdot N^2 \cdot K$$

μ is the permeability of the inductor core

K is a constant which depends on the inductor geometry

N is the number of turns of wire

Units: henry = $\frac{\text{volt} \cdot \text{sec}}{\text{amp}}$

mH = $10^{-3} \cdot \text{H}$

$\mu\text{H} = 10^{-6} \cdot \text{H}$

Basic equations you should know:

$$v_L = L \frac{d}{dt} i_L$$

$$i_L = \frac{1}{L} \int_{-\infty}^t v_L dt = \frac{1}{L} \int_0^t v_L dt + i_L(0)$$

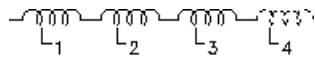
/ initial current

Inductor current **cannot** change instantaneously

Energy stored in electric field: $W_L = \frac{1}{2} \cdot L \cdot I_L^2$

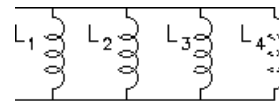
series:

$$L_{eq} = L_1 + L_2 + L_3 + \dots$$



parallel:

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots}$$

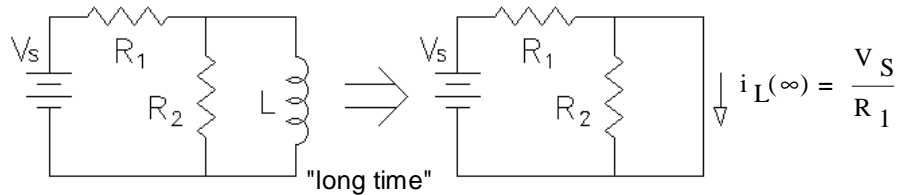


Steady-state of Final conditions

If a circuit has been connected for "a long time", then it has reached a steady state condition. that means the currents and voltages are no longer changing.

$$\frac{d}{dt} i_L = 0 \quad v_L = L \frac{d}{dt} i_L = 0$$

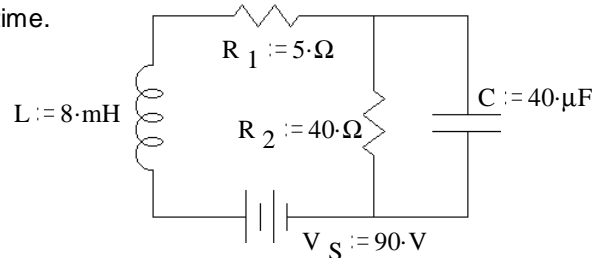
no voltage means it looks like a short



13. The following circuit has been connected as shown for a long time.

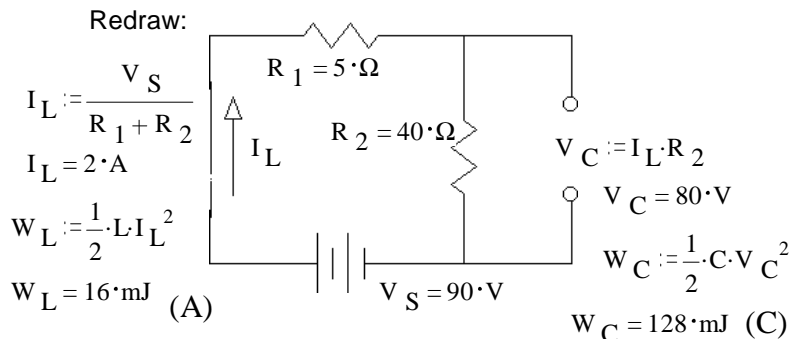
The energy stored in the inductor is:

- (A) 0.016·J
- (B) 0.020·J
- (C) 1.3·J
- (D) 1.6·J



14. The energy stored in the capacitor is:

- (A) 0.002·J
- (B) 0.041·J
- (C) 0.128·J
- (D) 0.162·J



Complex Numbers

Rectangular Form

$j = \sqrt{-1}$ the imaginary number

$$\mathbf{A} = a + b \cdot j$$

$$\text{Re}(\mathbf{A}) = a \quad \text{Im}(\mathbf{A}) = b$$

Polar Form

$$\mathbf{A} = A \cdot e^{j\theta}$$

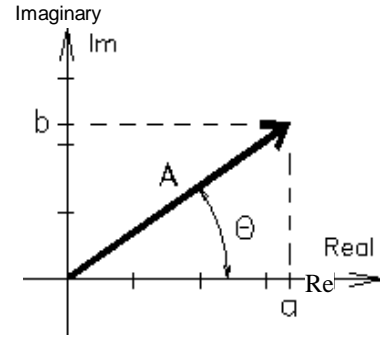
$$\text{Re}(\mathbf{A}) = A \cdot \cos(\theta) \quad \text{Im}(\mathbf{A}) = A \cdot \sin(\theta)$$

Conversions

$$A = |\mathbf{A}| = \sqrt{a^2 + b^2} \quad \theta = \arg(\mathbf{A}) = \text{atan}\left(\frac{b}{a}\right)$$

$$a = A \cdot \cos(\theta) \quad b = A \cdot \sin(\theta)$$

$$\mathbf{A} = A \cdot e^{j\theta} = A \cdot \cos(\theta) + A \cdot \sin(\theta) \cdot j$$



Special Cases

$$j := \sqrt{-1} = e^{j \cdot 90 \cdot \text{deg}} \quad \frac{1}{j} = -j = e^{-j \cdot 90 \cdot \text{deg}} \quad e^{j \cdot 0 \cdot \text{deg}} = 1 \quad e^{-j \cdot 180 \cdot \text{deg}} = e^{-j \cdot 180 \cdot \text{deg}} = -1$$

$$j \cdot e^{j\theta} = e^{j \cdot (\theta + 90 \cdot \text{deg})}$$

Define a 2nd number: rect: $\mathbf{D} = c + d \cdot j$ polar: $\mathbf{D} = D \cdot e^{j\phi}$

Equality

$\mathbf{A} = \mathbf{D}$ if and only if $a = c$ and $b = d$ OR $A = D$ and $\theta = \phi$

Addition and Subtraction

$$\mathbf{A} + \mathbf{D} = (a + b \cdot j) + (c + d \cdot j) = (a + c) + (b + d) \cdot j$$

$$\mathbf{A} - \mathbf{D} = (a + b \cdot j) - (c + d \cdot j) = (a - c) + (b - d) \cdot j$$

Convert polars to rectangular form first

Multiplication and Division

$$\mathbf{A} \cdot \mathbf{D} = (a + b \cdot j) \cdot (c + d \cdot j) = (a \cdot c - b \cdot d) + (b \cdot c + a \cdot d) \cdot j$$

Rectangular:

$$\frac{\mathbf{A}}{\mathbf{D}} = \frac{a + b \cdot j}{c + d \cdot j} = \frac{a + b \cdot j}{c + d \cdot j} \cdot \frac{c - d \cdot j}{c - d \cdot j} = \frac{a \cdot c + b \cdot d}{c^2 + d^2} + \frac{b \cdot c - a \cdot d}{c^2 + d^2} \cdot j$$

Polar:

$$\mathbf{A} \cdot \mathbf{D} = A \cdot e^{j\theta} \cdot D \cdot e^{j\phi} = A \cdot D \cdot e^{j \cdot (\theta + \phi)}$$

$$\frac{\mathbf{A}}{\mathbf{D}} = \frac{A \cdot e^{j\theta}}{D \cdot e^{j\phi}} = \frac{A}{D} \cdot e^{j \cdot (\theta - \phi)}$$

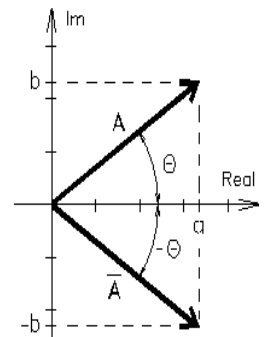
Powers

$$\mathbf{A}^n = A^n \cdot e^{j \cdot n \cdot \theta} = A^n \cdot \cos(n \cdot \theta) + A^n \cdot \sin(n \cdot \theta) \cdot j$$

Convert rectangulars first, usually

Conjugates

<u>complex number</u>	<u>Conjugate</u>	$\overline{\overline{\mathbf{A}}} = \mathbf{A}$
$\mathbf{A} = a + b \cdot j$	$\overline{\mathbf{A}} = a - b \cdot j$	
$\mathbf{A} = A \cdot e^{j\theta}$	$\overline{\mathbf{A}} = A \cdot e^{-j\theta}$	
$\mathbf{F} = \frac{3 + 5 \cdot j}{(2 - 6 \cdot j) \cdot e^{j \cdot 40 \cdot \text{deg}}}$	$\overline{\mathbf{F}} = \frac{3 - 5 \cdot j}{(2 + 6 \cdot j) \cdot e^{-j \cdot 40 \cdot \text{deg}}}$	



Euler's equation

$$e^{j\alpha} = \cos(\alpha) + j \cdot \sin(\alpha) \quad \text{OR:} \quad \cos(\alpha) = \frac{e^{j\alpha} + e^{-j\alpha}}{2} \quad \sin(\alpha) = \frac{e^{j\alpha} - e^{-j\alpha}}{2 \cdot j}$$

$$e^{j \cdot (\omega \cdot t + \theta)} = \cos(\omega \cdot t + \theta) + j \cdot \sin(\omega \cdot t + \theta)$$

$$\text{Re}\left[e^{j \cdot (\omega \cdot t + \theta)}\right] = \cos(\omega \cdot t + \theta)$$

If we freeze this at time $t=0$, then we can represent $\cos(\omega \cdot t + \theta)$ by $e^{j\theta}$

Calculus

Remember, when we write $e^{j\theta}$, we really mean $e^{j \cdot (\omega \cdot t + \theta)}$

$$\frac{d}{dt} \mathbf{A} = \frac{d}{dt} (A \cdot e^{j\theta}) = j \cdot \omega \cdot A \cdot e^{j\theta} = \omega \cdot A \cdot e^{j \cdot (\theta + 90 \cdot \text{deg})}$$

$$\int \mathbf{A} dt = \int A \cdot e^{j\theta} dt = \frac{1}{j \cdot \omega} \cdot A \cdot e^{j\theta} = \frac{1}{\omega} \cdot A \cdot e^{j \cdot (\theta - 90 \cdot \text{deg})}$$

Phasor analysis of Steady-State Sinusoidal AC

The math is all based on the Euler's equation

Euler's equation $e^{j\alpha} = \cos(\alpha) + j\sin(\alpha)$

$$\cos(\alpha) = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$$

OR:

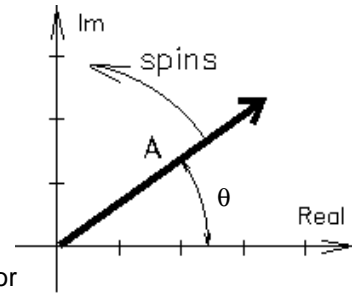
$$\sin(\theta) = \frac{e^{j\alpha} - e^{-j\alpha}}{2j}$$

$$e^{j(\omega t + \theta)} = \cos(\omega t + \theta) + j\sin(\omega t + \theta)$$

$$\text{Re}[e^{j(\omega t + \theta)}] = \cos(\omega t + \theta)$$

If we freeze this at time $t=0$, then we can represent $\cos(\omega t + \theta)$ by $e^{j\theta}$

That's the phasor



Phasors are drawn on a complex plane.

Phasor

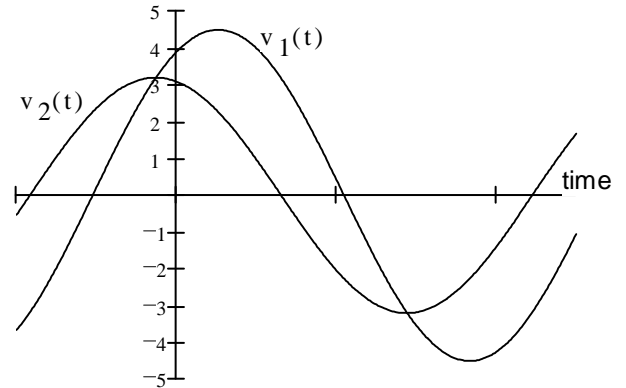
voltage: $v(t) = V_p \cdot \cos(\omega t + \phi)$ $V(\omega) = V_p \cdot e^{j\phi}$

current: $i(t) = I_p \cdot \cos(\omega t + \phi)$ $I(\omega) = I_p \cdot e^{j\phi}$

Phasors are used for adding and subtracting sinusoidal waveforms.

15. Add the sinusoidal voltages $v_1(t) = 4.5 \cdot V \cdot \cos(\omega t - 30\text{-deg})$
and $v_2(t) = 3.2 \cdot V \cdot \cos(\omega t + 15\text{-deg})$

- (A) $7.13 \cdot \cos(\omega t - 11.5\text{-deg}) \cdot V$
- (B) $7.13 \cdot \cos(\omega t - 45\text{-deg}) \cdot V$
- (C) $7.7 \cdot \cos(\omega t - 11.5\text{-deg}) \cdot V$
- (D) $7.7 \cdot \cos(\omega t - 15\text{-deg}) \cdot V$



using phasor notation, draw a phasor diagram of the three phasors, then convert back to time domain form.

$$v_1(t) = 4.5 \cdot V \cdot \cos(\omega t - 30\text{-deg})$$

$$V_1(\omega) = 4.5V \angle -30^\circ \quad \text{or:} \quad V_1(\omega) = 4.5 \cdot V \cdot e^{-j30\text{-deg}}$$

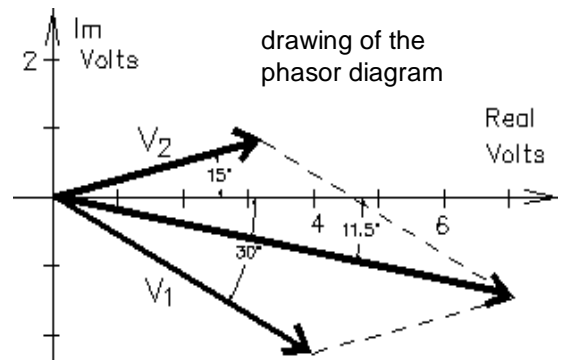
and $v_2(t) = 3.2 \cdot V \cdot \cos(\omega t + 15\text{-deg})$

$$V_2(\omega) = 3.2V \angle 15^\circ \quad \text{or:} \quad V_2(\omega) = 3.2 \cdot V \cdot e^{j15\text{-deg}}$$

I'm going to drop the (ω) notation from the phasor notation, it gets cumbersome, but remember that phasors are in the frequency domain..

$$V_1 = 4.5V \angle -30^\circ \quad \text{or:} \quad V_1 := 4.5 \cdot V \cdot e^{-j30\text{-deg}}$$

$$V_2 = 3.2V \angle 15^\circ \quad \text{or:} \quad V_2 := 3.2 \cdot V \cdot e^{j15\text{-deg}}$$



Add like vectors, first change to the rectangular form

$V_1 = 4.5V \angle -30^\circ$	$4.5 \cdot V \cdot \cos(-30\text{-deg}) = 3.897 \cdot V$	$4.5 \cdot V \cdot \sin(-30\text{-deg}) = -2.25 \cdot V$	$V_1 = 3.897 - 2.25j \cdot V$	\ } add
$V_2 = 3.2V \angle 15^\circ$	$3.2 \cdot V \cdot \cos(15\text{-deg}) = 3.091 \cdot V$	$3.2 \cdot V \cdot \sin(15\text{-deg}) = 0.828 \cdot V$	$V_2 = 3.091 + 0.828j \cdot V$	
	Add real parts:	$3.897 + 3.091 = 6.988$	$V_3 := V_1 + V_2$	
	Add imaginary parts:	$-2.25 + 0.828 = -1.422$	$V_3 = 6.988 - 1.422j \cdot V$	sum

Change V_3 back to polar coordinates:

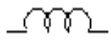
$$\sqrt{6.988^2 + 1.422^2} = 7.131 \quad \text{atan}\left(\frac{-1.422}{6.988}\right) = -11.502 \cdot \text{deg}$$

Change V_3 back to the time domain:

$$v_3(t) = v_1(t) + v_2(t) = 7.13 \cdot \cos(\omega t - 11.5\text{-deg}) \cdot V \quad \text{(A)}$$

Impedance (like resistance)

Inductor



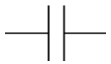
$$v_L = L \cdot \frac{d}{dt} i_L = L \cdot \frac{d}{dt} I_p \cdot e^{j(\omega t + \theta)} = j \cdot \omega \cdot L \cdot [I_p \cdot e^{j(\omega t + \theta)}]$$

in phasor notation ----> $V_L(\omega) = j \cdot \omega \cdot L \cdot I(\omega)$

AC impedance

$$Z_L = j \cdot \omega \cdot L$$

Capacitor



$$i_C = C \cdot \frac{d}{dt} v_C = C \cdot \frac{d}{dt} V_p \cdot e^{j(\omega t + \theta)} = j \cdot \omega \cdot C \cdot [V_p \cdot e^{j(\omega t + \theta)}]$$

in phasor notation ----> $I_C(\omega) = j \cdot \omega \cdot C \cdot V(\omega)$

$$V_C(\omega) = \frac{1}{j \cdot \omega \cdot C} \cdot I(\omega)$$

$$Z_C = \frac{1}{j \cdot \omega \cdot C} = \frac{-j}{\omega \cdot C}$$

Resistor



$$v_R = i_R \cdot R$$

$$V_R(\omega) = R \cdot I(\omega)$$

$$Z_R = R$$

You can use impedances just like resistances as long as you deal with the complex arithmetic.
ALL the DC circuit analysis techniques will work with AC.

series:

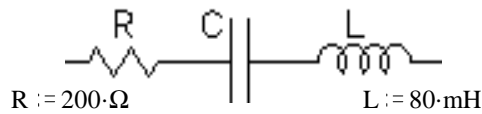


$$Z_{eq} = Z_1 + Z_2 + Z_3 + \dots$$

Example:

$$f := 500 \text{ Hz}$$

$$\omega := 2 \cdot \pi \cdot f = \omega = 3141.6 \cdot \frac{\text{rad}}{\text{sec}}$$



$$R := 200 \cdot \Omega$$

$$C := 0.6 \cdot \mu\text{F}$$

$$L := 80 \text{ mH}$$

$$j \cdot \omega \cdot L = 251.327j \cdot \Omega$$

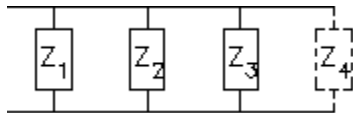
$$\frac{1}{j \cdot \omega \cdot C} = -530.516j \cdot \Omega$$

$$Z_{eq} := R + \frac{1}{j \cdot \omega \cdot C} + j \cdot \omega \cdot L = 200 \cdot \Omega - 530.5j \cdot \Omega + 251.3j \cdot \Omega = 200 - 279.2j \cdot \Omega \quad \text{rectangular form}$$

$$\sqrt{(200 \cdot \Omega)^2 + (279.2 \cdot \Omega)^2} = 343.4 \cdot \Omega \quad \text{atan}\left(\frac{-279.2 \cdot \Omega}{200 \cdot \Omega}\right) = -54.38 \cdot \text{deg}$$

$$Z_{eq} = 343.4 \Omega / -54.4^\circ \quad \text{polar form}$$

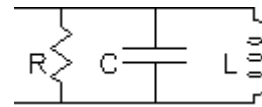
parallel:



$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \dots}$$

Example: Same parts and frequency as above

$$f := 500 \text{ Hz} \quad \omega := 2 \cdot \pi \cdot f \quad \omega = 3141.6 \cdot \frac{\text{rad}}{\text{sec}}$$



$$R := 200 \cdot \Omega$$

$$C := 0.6 \cdot \mu\text{F}$$

$$L := 80 \text{ mH}$$

$$\omega \cdot C = 1.885 \cdot 10^{-3} \cdot \frac{1}{\Omega}$$

$$Z_{eq} := \frac{1}{\frac{1}{R} + \frac{1}{\left(\frac{1}{j \cdot \omega \cdot C}\right)} + \frac{1}{j \cdot \omega \cdot L}} = \frac{1}{\frac{1}{R} + j \cdot \omega \cdot C - \frac{j}{\omega \cdot L}} = \frac{1}{\frac{1}{200 \cdot \Omega} + 1.885 \cdot 10^{-3} \cdot j - 3.979 \cdot 10^{-3} \cdot j \cdot \frac{1}{\Omega}}$$

$$= \frac{1}{\left(5 \cdot 10^{-3} - 2.094 \cdot 10^{-3} \cdot j\right) \cdot \frac{1}{\Omega} \cdot \left(5 \cdot 10^{-3} + 2.094 \cdot 10^{-3} \cdot j\right)} = \frac{1}{2.93848 \cdot 10^{-5}} = 170.156 + 71.261j \cdot \Omega$$

If you want the answer in polar form, it's easier to convert the denominator first.

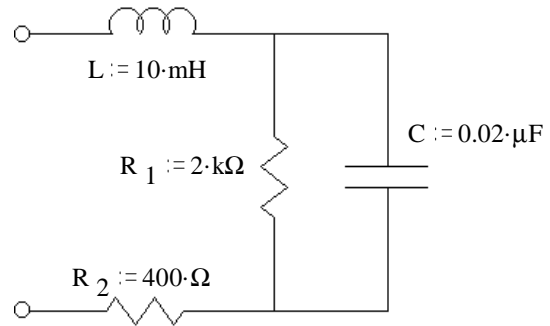
$$\sqrt{\left(5 \cdot 10^{-3} \cdot \frac{1}{\Omega}\right)^2 + \left(2.094 \cdot 10^{-3} \cdot \frac{1}{\Omega}\right)^2} = 5.4 \cdot 10^{-3} \cdot \frac{1}{\Omega} \quad \text{atan}\left(\frac{2.094 \cdot 10^{-3} \cdot \Omega}{5 \cdot 10^{-3} \cdot \Omega}\right) = 22.72 \cdot \text{deg}$$

$$\frac{1}{5.4 \cdot 10^{-3} \cdot \frac{1}{\Omega}} = 185.185 \cdot \Omega$$

$$Z_{eq} = 185.2 / 22.7^\circ$$

16. Find Z_{eq} . $f := 3 \cdot \text{kHz}$

- (A) $Z_{eq} = 442 \angle -25.2^\circ \Omega$
- (B) $Z_{eq} = 2.03 \angle 34.5^\circ \text{ k}\Omega$
- (C) $Z_{eq} = 1.84 \angle -24.8^\circ \text{ k}\Omega$
- (D) $Z_{eq} = 3.44 \angle -45.8^\circ \text{ k}\Omega$



find ω $\omega := 2 \cdot \pi \cdot f$ $\omega = 1.885 \cdot 10^4 \cdot \frac{\text{rad}}{\text{sec}}$

$$Z_{eq} := j \cdot \omega \cdot L + \frac{1}{\frac{1}{R_1} + j \cdot \omega \cdot C} + R_2$$

$$\frac{1}{R_1} + j \cdot \omega \cdot C = 5 \cdot 10^{-4} + 3.77 \cdot 10^{-4} j \quad \cdot \frac{1}{\Omega}$$

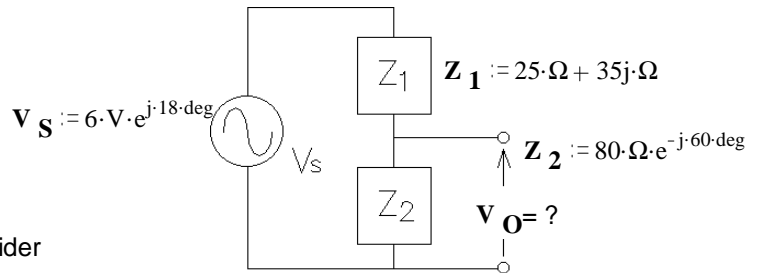
divide $\frac{1}{\frac{1}{R_1} + j \cdot \omega \cdot C} = 1.275 \cdot 10^3 - 961.412 j \quad \cdot \Omega$

add $R_2 + j \cdot \omega \cdot L = 400 + 188.496 j \quad \cdot \Omega$ $Z_{eq} = 1.675 \cdot 10^3 - 772.917 j \quad \cdot \Omega$

rect to polar $\sqrt{1675^2 + 772.9^2} = 1844.7$ $\text{atan}\left(\frac{-772.9}{1675}\right) = -24.8 \cdot \text{deg}$ (C)

17. Find V_O in the circuit shown.

- (A) $3.91 \cdot V \cdot e^{j \cdot 19.9 \cdot \text{deg}}$
- (B) $3.91 \cdot V \cdot e^{-j \cdot 100 \cdot \text{deg}}$
- (C) $6.53 \cdot V \cdot e^{-j \cdot 69.8 \cdot \text{deg}}$
- (D) $6.53 \cdot V \cdot e^{-j \cdot 14.2 \cdot \text{deg}}$



$$V_O := \frac{Z_2}{Z_1 + Z_2} \cdot V_S \quad \text{Simple voltage divider}$$

$$|Z_2| \cdot \cos(-60 \cdot \text{deg}) = 40 \cdot \Omega \quad |Z_2| \cdot \sin(-60 \cdot \text{deg}) = -69.282 \cdot \Omega \quad Z_2 = 40 - 69.282 j \quad \cdot \Omega$$

$$Z_1 + Z_2 = 25 \cdot \Omega + 35 j \cdot \Omega + 40 \cdot \Omega - 69.282 j \cdot \Omega = 65 - 34.282 j \quad \cdot \Omega = 73.486 \cdot \Omega \cdot e^{-j \cdot 27.81 \cdot \text{deg}}$$

$$V_O := \frac{Z_2}{Z_1 + Z_2} \cdot V_S = \frac{80 \cdot \Omega \cdot e^{-j \cdot 60 \cdot \text{deg}}}{73.486 \cdot \Omega \cdot e^{-j \cdot 27.81 \cdot \text{deg}}} \cdot (6 \cdot V \cdot e^{j \cdot 18 \cdot \text{deg}}) = \frac{80 \cdot \Omega}{73.486 \cdot \Omega} \cdot 6 \cdot V \cdot e^{j \cdot (-60 - (-27.81) + 18) \cdot \text{deg}}$$

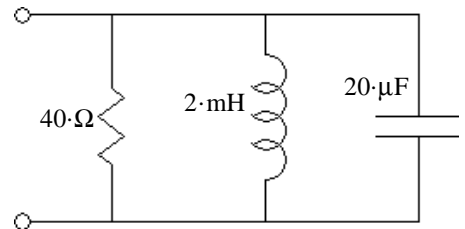
$$= 6.53 \cdot V \cdot e^{-j \cdot 14.2 \cdot \text{deg}} \quad \text{(D)}$$

18. The resonant frequency of the circuit shown is most nearly:

- (A) 5 rad/s
- (B) 20 rad/s
- (C) 5000 rad/s
- (D) 20000 rad/s

Resonance:

$$\omega_o = \frac{1}{\sqrt{L_{eq} \cdot C_{eq}}} = \frac{1}{\sqrt{2 \cdot \text{mH} \cdot 20 \cdot \mu\text{F}}} = 5000 \cdot \frac{\text{rad}}{\text{sec}} \quad \text{(C)}$$



19. What is the magnitude of the equivalent impedance of the circuit at the resonant frequency?

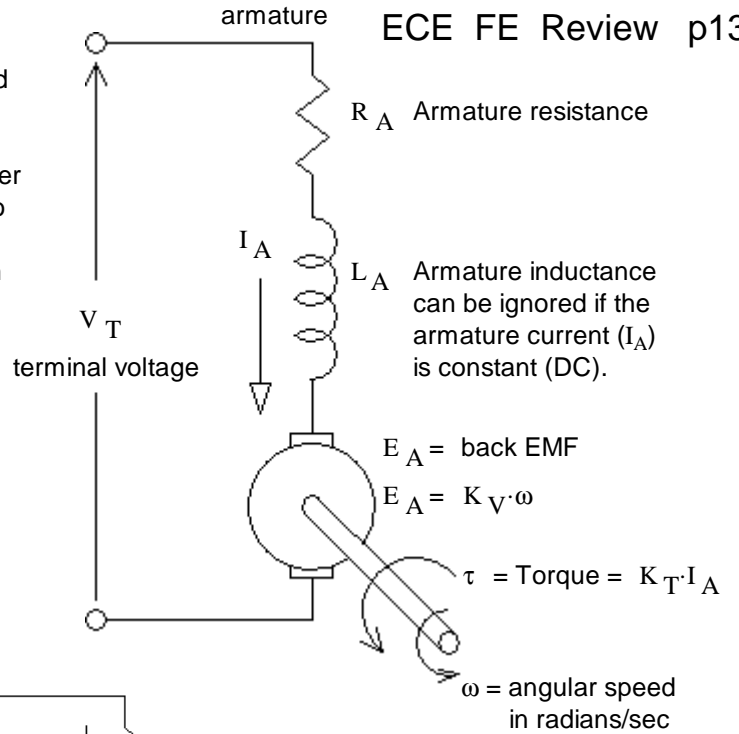
- (A) 0 Ohms
- (B) 4.44 Ohms
- (C) 4.961 Ohms
- (D) 40 Ohms

$$\frac{1}{j \cdot \omega_o \cdot L} = \frac{1}{0} = \infty \quad \text{(D)}$$

DC Motor

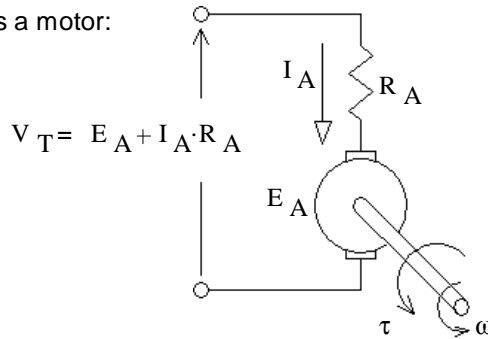
A simple DC motor consists of a rotating armature placed in a stationary magnetic field. The DC current flows through brushes which make contact with a commutator on the rotating armature. The commutator has a number of contact points and is wired to the armature windings so that its magnetic field is out of alignment with the stationary field. As the motor armature turns to align with the magnetic field, the connection is changed so that the next winding gets the current and the armature field is again out of alignment. The motor torque (τ) is directly proportional to the armature current (I_A) by the torque constant (K_T).

Because the armature windings are turning within a magnetic field, a voltage is induced on those windings. This voltage is known as the back EMF and opposes the armature current flow. The back EMF (E_A) is directly proportional to the armature angular velocity (ω) by the voltage constant (K_V). An electrical model of a DC motor is shown at right.



The stationary field may be created by permanent magnets or by a field winding.

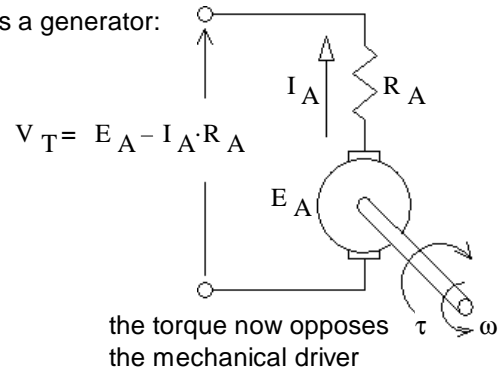
As a motor:



$K_T = \text{Torque constant}$
 $K_V = \text{Voltage constant}$
 $K_T = K_V$

If the shaft is turned faster by some external driver, the generated EMF could be larger than the terminal voltage. In that case the current reverses direction and the motor becomes a DC generator

As a generator:



20. A DC motor is producing a torque of 30 Nm. The current of the armature is halved. What is the approximate new torque? Ignore the armature resistance.

- (A) 15·N·m
- (B) 20·N·m
- (C) 30·N·m
- (D) 35·N·m

$$30 \cdot \text{N} \cdot \text{m} \cdot \frac{1}{2} = 15 \cdot \text{N} \cdot \text{m} \quad (\text{A})$$

21. A DC motor operates from 24 V (V_T) and has an armature resistance of 0.3Ω. At full load, the armature current is 10 A and the speed is 1200 rpm. Ignoring all losses except the armature resistance, find the no-load speed of the motor. Note: you may assume $I_A = 0$ when the motor is not loaded.

- (A) 1200·rpm
- (B) 1370·rpm
- (C) 1200·rpm
- (D) 2400·rpm

$$V_T := 24 \cdot \text{V} \quad R_A := 0.30 \cdot \Omega \quad I_A := 10 \cdot \text{A} \quad n := 1200 \cdot \text{rpm}$$

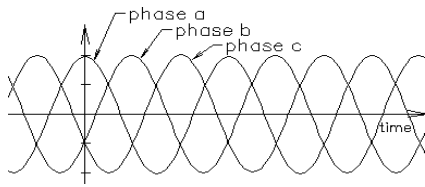
$$E_{Afl} := V_T - R_A \cdot I_A \quad E_{Afl} = 21 \cdot \text{V}$$

$$\text{Now } E_A := V_T \quad n = \frac{E_A}{E_{Afl}} \cdot (1200 \cdot \text{rpm}) = 1371 \cdot \text{rpm} \quad (\text{B})$$

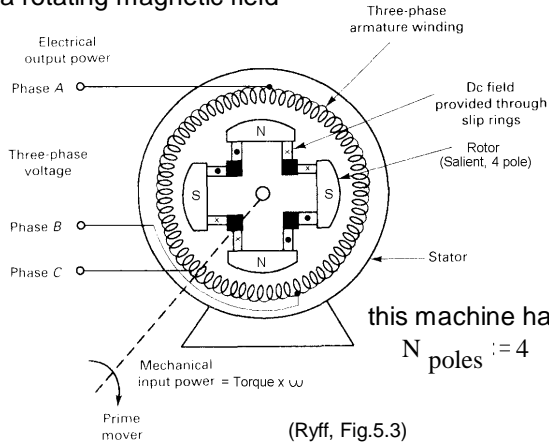
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Synchronous Generators & Motors

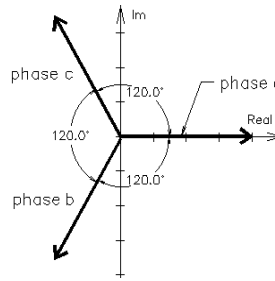
Synchronous machines run on and generate 3-phase power



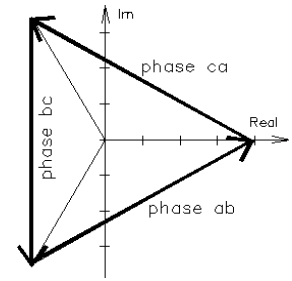
The 3 phases are wired in the stator to produce a rotating magnetic field



Wye connection:



Delta connection:



rotational speed = n_m or $n_{sync} = \frac{f \cdot \frac{2 \cdot \text{poles}}{cyc} \cdot \frac{60 \cdot \text{sec}}{min}}{N \text{ poles}} = \frac{7200 \cdot \text{rpm}}{\text{poles}}$ for 60Hz systems

$\omega = \frac{4 \cdot \pi \cdot f}{N \text{ poles}}$ $n_{sync} = \text{the synchronous speed}$

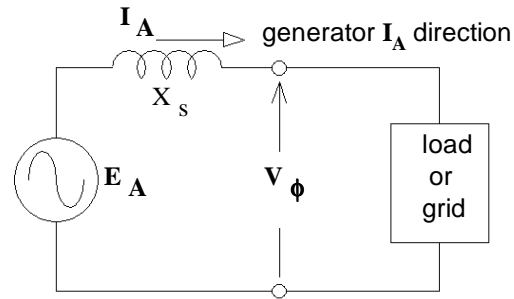
The rotor is an electromagnet with a DC field current. It will rotate along with the rotating magnetic field at the synchronous speed.

When spinning, the induced armature voltages (E_A for our phase) depends on the field current, I_f . I_f causes the field flux (called **excitation**).

Electrical analysis on a per-phase basis

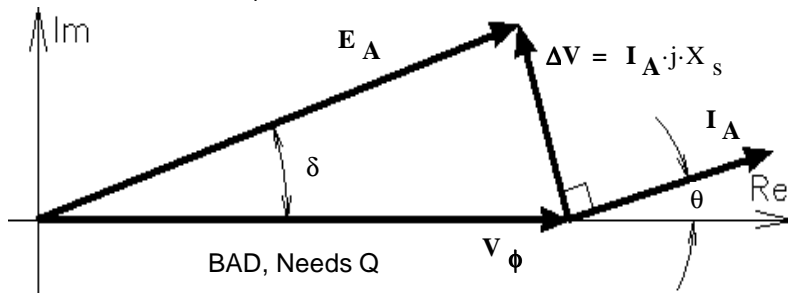
The electrical model of an armature winding:

X_s is the armature inductance (armature windings and leakage) (magnetization)

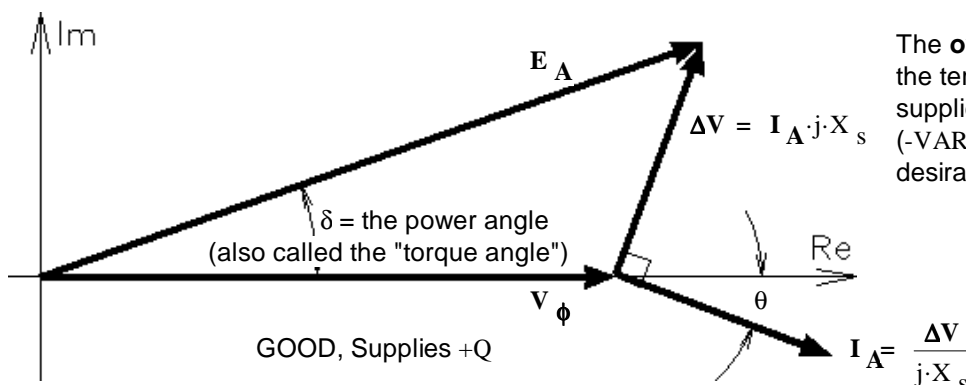


Synchronous Generators

$$E_A = I_A \cdot j \cdot X_s + V_\phi$$



The **under-excited** condition, the current leads the terminal voltage, V_ϕ . The generator supplies -Q (-VARs), that is, it absorbs +Q (+VARs), just like an inductive load. Usually not desirable.



The **over-excited** condition, the current lags the terminal voltage, V_ϕ . The generator supplies +Q (+VARs), that is, it absorbs -Q (-VARs), just like a capacitive load. Usually desirable.

Note: if δ reaches 90° , the generator will lose synchronization.