Electrical Review Lecture
Fundamentals of Engineering (FE)

Arn Stolp
Exam Information downloaded 4/14
from: www.ncees.org

You can download exam specs, and a reference manual at ncees.org. You can also register for the exam, take a practice test, and pay for it all. The computer-based FE exam costs $225 and takes 6 hours. This includes a tutorial, 5 hours and 20 minutes to answer 110 questions, and a 25 minute break. That's about 3 minutes per question. All questions are multiple choice and discipline-specific to the student's major. General knowledge topics, such as mathematics, appear on exams for all disciplines. You will take the FE exam on a 24 inch split-screen computer monitor. Half the screen will show the exam questions, and half will display the reference manual. The manual will be a searchable PDF. If you aren't sure of an answer to a question, you can bookmark it and return later, but don't leave it blank in the end. Guessing doesn't hurt you. Because you have to move through the exam quickly, FE questions are designed to be answered quickly. Many of them cover simple conceptual information. Read the problem carefully and ask yourself, "What's the concept they're testing for?"

Search online for information and YouTube videos to review for specific areas of the exam.

Calculator policy (2014) http://ncees.org/exams/calculator-policy/
Casio: All fx-115 models. Any Casio calculator must contain fx-115 in its model name.
Examples of acceptable Casio fx-115 models include (but are not limited to):
fx-115 MS fx-115 MS Plus fx-115 MS SR fx-115 ES fx-115 ES Plus
Hewlett Packard: The HP 33s and HP 35s models, but no others.
Texas Instruments: All TI-30X and TI-36X models. Any Texas Instruments calculator must contain either TI-30X or TI-36X in its model name. Examples of acceptable TI-30X and TI-36X models include (but are not limited to):
TI-30Xa TI-30Xa SOLAR TI-30Xa SE TI-30XS Multiview TI-30X IIB TI-30X IIS
TI-36X II TI-36X SOLAR TI-36X Pro

Civil: No electrical questions.
Chemical: Basics (3D), Process control (14)
Other Disciplines: Basics (14), Instrumentation (4)
Mechanical: Basics with motors and generators (6), Instrumentation (14)
Electrical: download "FE-Ele-CBT-specs_with-ranges.pdf" from www.ncees.org

This Lecture will touch on the subjects below:
MECHANICAL CBT Exam Specifications

	# of quest
6. Electricity and Magnetism	**3–5**
A. Charge, current, voltage, power, and energy	
B. Current and voltage laws (Kirchhoff, Ohm)	
C. Equivalent circuits (series, parallel)	
D. AC circuits	
E. Motors and generators	

Answers to problems in following pages

1. (D) 2. (A) 3. (B) 4. (A) 5. (D) 6. (D) 7. (A) 8. (C) 9. (B) 10. (D) 11. (C)
12. (B) 13. (A) 14. (C) 15. (A) 16. (C) 17. (D) 18. (C) 19. (D) 20. (A) 21. (B)

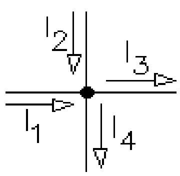
Electrical Engineering FE Review Lecture

A. Stolp
4/24/15

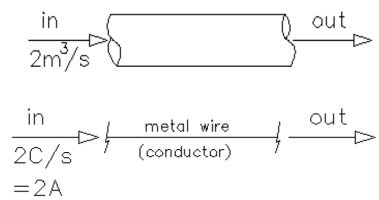
Basic electrical quantities	**Letter used**	**Units**	**Fluid Analogy**
Charge, actually moves	Q	Coulomb (C)	m^3
Current, like fluid flow	$I = \frac{Q}{sec}$	Amp (A, mA, μA,...)	$\frac{m^3}{sec}$
Voltage, like pressure	V or E	volt (V, mV, kV,...)	$Pa = 1 \cdot \frac{N}{m^2}$
Resistance	$R = \frac{V}{I}$	Ohm (Ω, kΩ, MΩ,...)	
Conductance	$G = \frac{1}{R}$	Siemens (S, also mho, old unit)	
Power = energy/time	$P = V \cdot I$	Watt (W, mW, kW, MW,...)	W

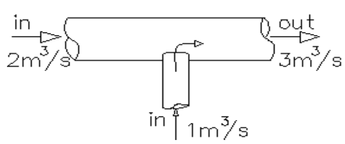
KCL, **K**irchhoff's **C**urrent **L**aw

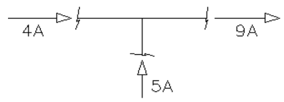
$I_{in} = I_{out}$ of any point, part, or section

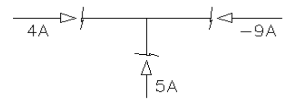


$$I_{in} = I_{out}$$
$$I_1 + I_2 = I_3 + I_4$$

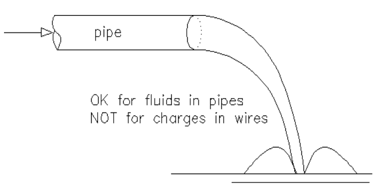






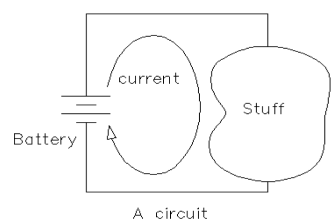


negative current means the direction arrow is wrong



Conductors vs Nonconductors

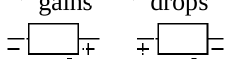
Battery also obeys KCL. No accumulation of charge anywhere, so it must circulate around. Leads to the concept of a "Circuit"



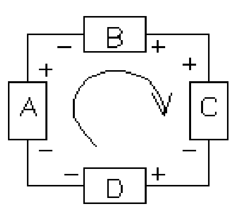
Voltage is like pressure

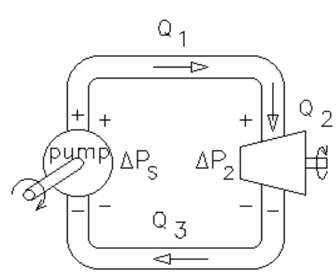
KVL, **K**irchhoff's **V**oltage **L**aw

$$V_{gains} = V_{drops}$$



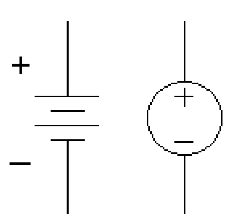
around any loop



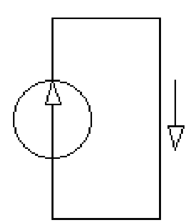


Sources

Ideal batteries or voltage sources always make the same voltage regardless of current.



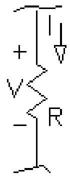
Ideal current sources always make the same current flow regardless of the voltage



Must have a path for the current to flow

Resistors

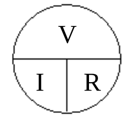
Ohm's law



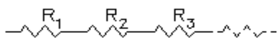
$$I = \frac{V}{R}$$

$$R = \frac{V}{I}$$ definition of resistance and the unit "Ω"

Ideal wires have no resistance



series:



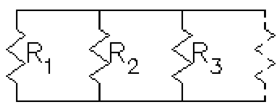
$$R_{eq} = R_1 + R_2 + R_3 + \ldots$$

Exactly the **same current** through each resistor

Voltage divider:

$$V_{Rn} = V_{total} \cdot \frac{R_n}{R_1 + R_2 + R_3 + \ldots}$$

parallel: $$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots}$$



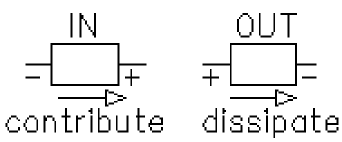
Exactly the **same voltage** across each resistor

current divider:

$$I_{Rn} = I_{total} \cdot \frac{\frac{1}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots}$$

Power

$P_{IN} = P_{OUT}$ for resistor circuits

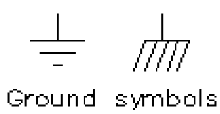


$P = V \cdot I$ for everything

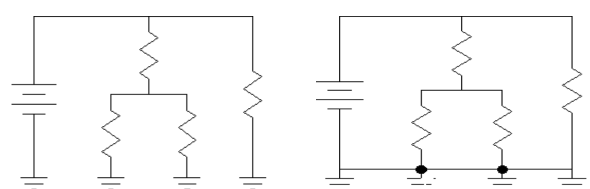
$= I^2 \cdot R = \frac{V^2}{R}$ for resistors

Energy (Joules) = Power X time

Ground

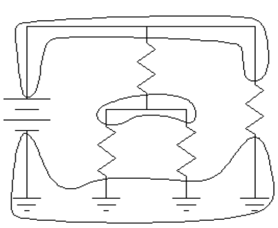


Ground is considered zero volts and is a reference for other voltages.



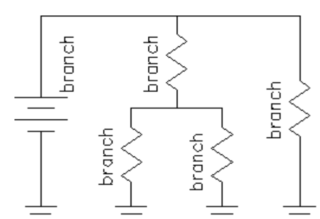
Nodes & Branches

Node = all points connected by wire, all at same voltage (potential)

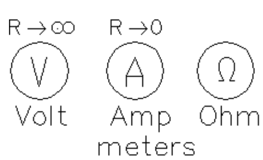


ground is a node

Branch = all parts with the same current



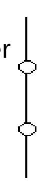
Meters



ideally: voltmeter

open

ammeter

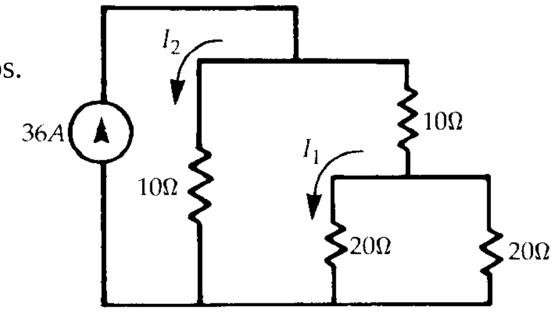


short

Examples

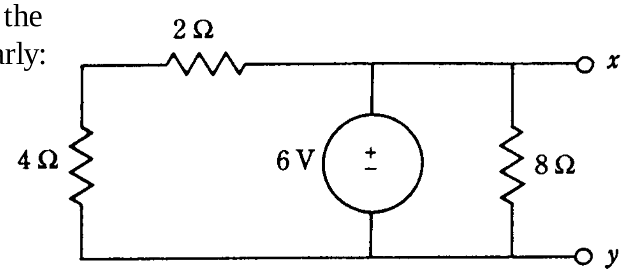
1. Find I_2 in amps.

   (A) 9
   (B) 12
   (C) 18
   (D) 24
   (E) 27



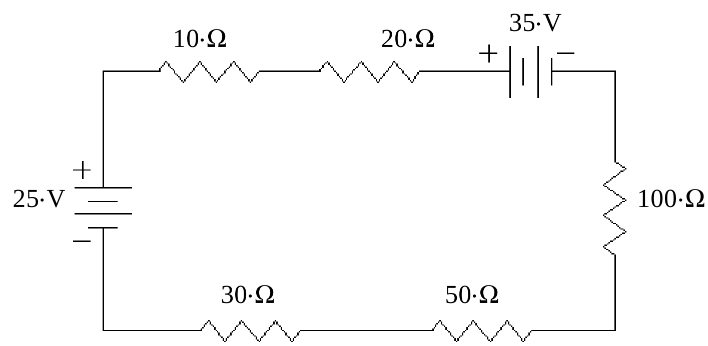
2. If a 12-ohm resistor is connected across terminals *xy* in the circuit shown, the current through it would be most nearly:

   (A) 0.5 A
   (B) 1.25 A
   (C) 2.0 A
   (D) 2.25 A
   (E) 5.75 A



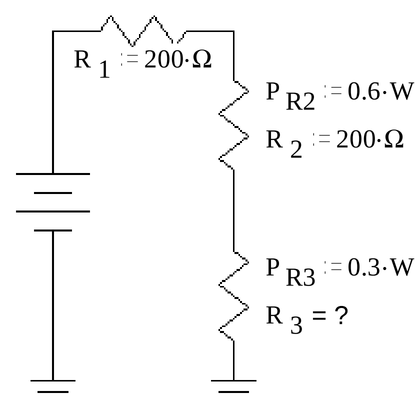
3. The voltage across the 50-ohm resistor in the circuit shown is most nearly:

   (A) 1.43 V
   (B) 2.4 V
   (C) 5.95 V
   (D) 8.33 V
   (E) 8.57 V



4. In the circuit shown, the power loss in R_2 is 0.6 W and the power loss in R_3 is 0.3 W. What is the value of the resistor, R_3?

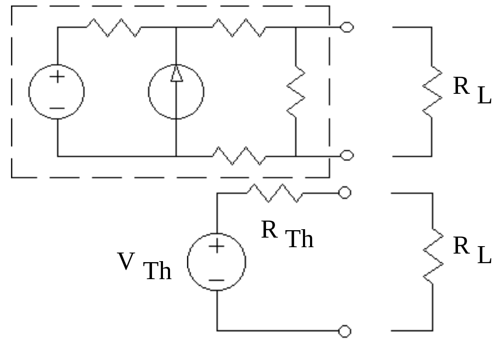
   (A) 100 Ω
   (B) 141 Ω
   (C) 283 Ω
   (D) 400 Ω



5. In the circuit above, what is output power of the battery?

   (A) 0.6 W
   (B) 0.9 W
   (C) 1.2 W
   (D) 1.5 W

Thévevin Equivalent Circuit



A simplified model can be used for any combination of sources and resistors.

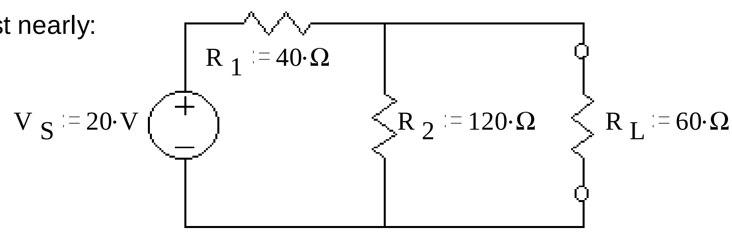
To calculate a circuit's Thévenin equivalent:
1) Remove the load and calculate the open-circuit voltage where the load used to be. This is the Thévenin voltage (V_{Th}).

2) Zero all the sources. (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)

3) Compute the total resistance between the load terminals. (DO NOT include the load in this resistance.) This is the Thévenin source resistance (R_{Th}).

4) Draw the Thévenin equivalent circuit and add your values.

6. The Thévenin voltage (V_{Th}) of the circuit shown is most nearly:



(A) 0 V
(B) 2.4 V
(C) 5 V
(D) 15 V
(E) 20 V

7. The Thévenin resistance (R_{Th}) of the circuit above is most nearly:

(A) 30 Ω
(B) 40 Ω
(C) 120 Ω
(D) 160 Ω

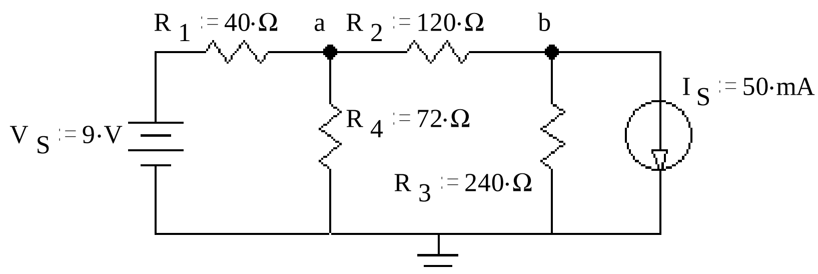
8. The Norton current (I_N) of the circuit above is most nearly:

(A) 0.125 A
(B) 0.25 A
(C) 0.5 A
(D) 0.67 A

Nodal Analysis Steps

1) If the circuit doesn't already have a ground, label one node as ground (zero voltage).
   If the ground can be defined as one side of a voltage source, that will make the following steps easier.
   Label the remaining node, either with known voltages or with letters, a, b, ...
2) Label unknown node voltages as V_a, V_b, ... and label the current in each resistor as I_1, I_2, ...
3) Write Kirchoff's current equations for each unknown node.
4) Replace the currents in your **KCL** equations with expressions like this. $\dfrac{V_a - V_b}{R_1}$ Ohm's law relationship using the nodal voltages.
5) Solve the multiple equations for the multiple unknown voltages.

9. The nodal equation for node b is:



(A) $\dfrac{9\cdot V - V_a}{40\cdot\Omega} = \dfrac{V_a - V_b}{120\cdot\Omega} + \dfrac{V_a}{72\cdot\Omega}$

(B) $\dfrac{V_a - V_b}{120\cdot\Omega} = \dfrac{V_b}{240\cdot\Omega} + 50\cdot mA$

(C) $\dfrac{V_b - V_a}{120\cdot\Omega} = \dfrac{V_b}{240\cdot\Omega} + 50\cdot mA$

(D) $\dfrac{9\cdot V - V_a}{40\cdot\Omega} - \dfrac{V_a}{72\cdot\Omega} + \dfrac{V_a - V_b}{120\cdot\Omega} = \dfrac{V_b - 0\cdot V}{240\cdot\Omega} + 50\cdot mA$

Sinusoidal AC

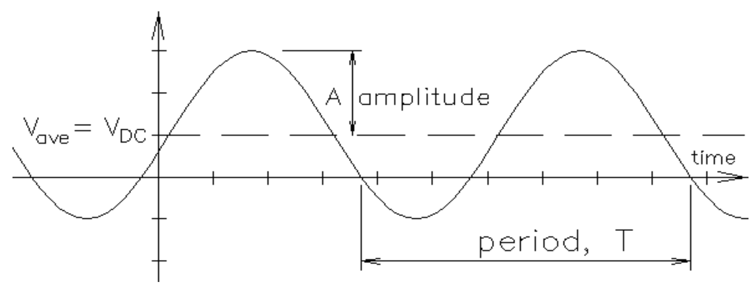
T = Period = repeat time

f = frequency, cycles / second $\quad f = \dfrac{1}{T} = \dfrac{\omega}{2\cdot\pi}$

ω = radian frequency, radians/sec $\quad \omega = 2\cdot\pi\cdot f$

A = amplitude

DC = average $\quad$ RMS: $\dfrac{A}{\sqrt{2}}$

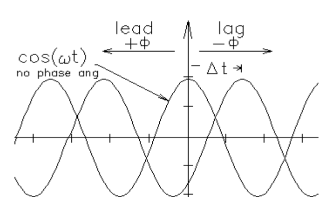


$y(t) = A\cdot\cos(\omega\cdot t + \phi)$

voltage: $v(t) = V_p\cdot\cos(\omega\cdot t + \phi)$ $\quad$ current: $i(t) = I_p\cdot\cos(\omega\cdot t + \phi)$

Phase: $\phi = -\dfrac{\Delta t}{T}\cdot 360\cdot deg$ $\quad$ or: $\phi = -\dfrac{\Delta t}{T}\cdot 2\cdot\pi\cdot rad$

Phase:



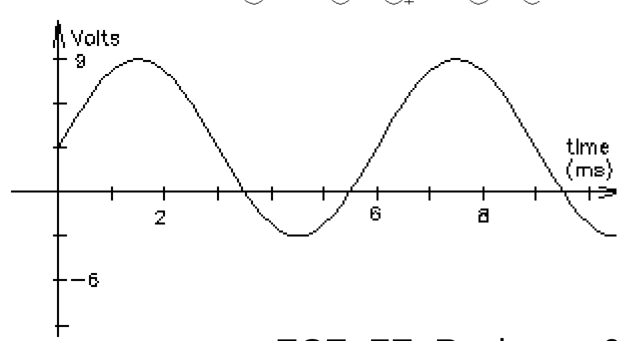
10. For the following waveforms, find v(t):

(A) $v(t) := 6\cdot V\cdot\cos(167\cdot t - 90\cdot deg) + 3\cdot V$

(B) $v(t) := 3\cdot V\cdot\cos(167\cdot t - 90\cdot deg) + 6\cdot V$

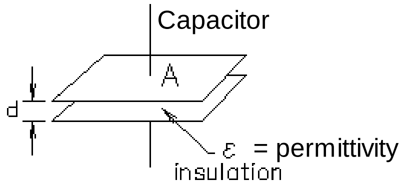
(C) $v(t) := 6\cdot V\cdot\cos(1047\cdot t + 90\cdot deg) + 3\cdot V$

(D) $v(t) := 6\cdot V\cdot\cos(1047\cdot t - 90\cdot deg) + 3\cdot V$

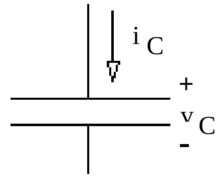


Capacitors

Electrical equivalent:

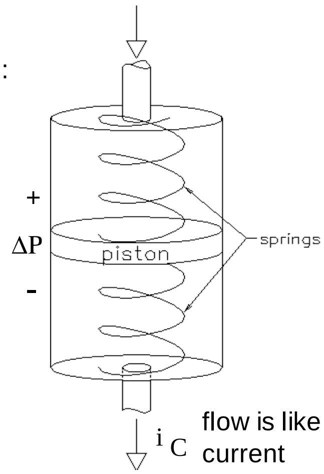


$$C = \varepsilon\cdot\frac{A}{d} = \frac{Q}{V} = \frac{dq}{dv}$$



like pressure ΔP

Fluid Model:



flow is like current

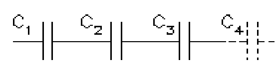
Basic equations you should know: $C = \frac{Q}{V}$ $\quad v_C = \frac{1}{C}\cdot\int_{-\infty}^{t} i_C\,dt = \frac{1}{C}\cdot\int_{0}^{t} i_C\,dt + v_C(0)$ (initial voltage) $\quad i_C = C\cdot\frac{d}{dt}v_C$

Units: $\text{farad} = \frac{\text{coul}}{\text{volt}} = \frac{\text{amp}\cdot\text{sec}}{\text{volt}}$ $\qquad \mu F = 1\cdot 10^{-6}\cdot\text{farad}$ $\qquad pF = 1\cdot 10^{-12}\cdot\text{farad}$

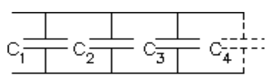
Capacitor voltage **cannot** change instantaneously

Energy stored in electric field: $W_C = \frac{1}{2}\cdot C\cdot V_C^{\,2}$

series: $C_{eq} = \dfrac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots}$

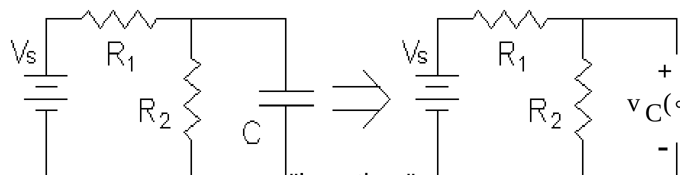


parallel: $C_{eq} = C_1 + C_2 + C_3 + \ldots$



Steady-state or Final conditions

If a circuit has been connected for "a long time", then it has reached a steady state condition. that means the currents and voltages are no longer changing.



$$v_C(\infty) = V_S\cdot\frac{R_2}{R_1+R_2}$$

"long time"

$$\frac{d}{dt}v_C = 0 \qquad i_C = C\cdot\frac{d}{dt}v_C = 0$$

no current means it looks like an open

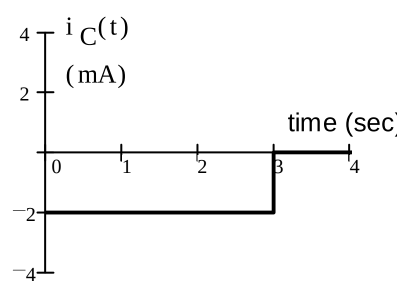
11. The current shown flows though a 50-µF capacitor.
    The initial voltage on the capacitor is 10V (Vc(0) = 10V).
    The capacitor voltage at 2.5 seconds is most nearly:

    (A) -110 V

    (B) -100 V

    (C) -90 V

    (D) 110 V



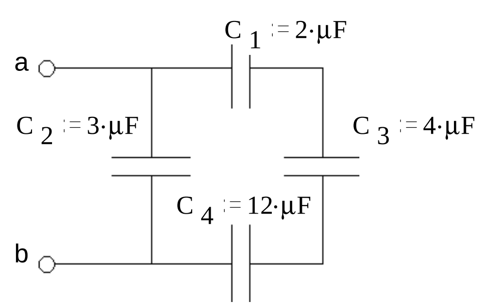
12. Find C_{eq} between terminals a and b.

    (A) 3.86 µF

    (B) 4.2 µF

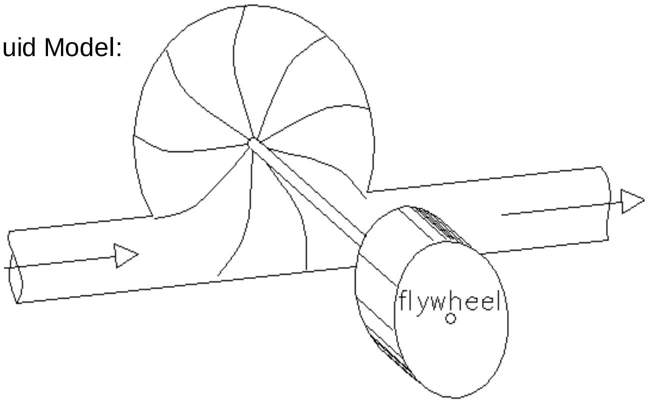
    (C) 4.67 µF

    (D) 21 µF

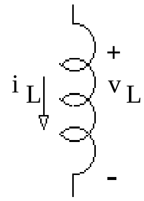


Inductors

Fluid Model:



Electrical equivalent:



$L = \mu_o \cdot N^2 \cdot K$

μ is the permeability of the inductor core

K is a constant which depends on the inductor geometry

N is the number of turns of wire

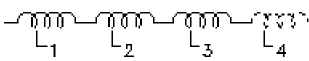
Units: $\text{henry} = \frac{\text{volt} \cdot \text{sec}}{\text{amp}}$ $\qquad$ $\text{mH} = 10^{-3} \cdot \text{H}$ $\qquad$ $\mu\text{H} = 10^{-6} \cdot \text{H}$

Basic equations you should know: $\quad v_L = L \cdot \frac{d}{dt} i_L \qquad i_L = \frac{1}{L} \cdot \int_{-\infty}^{t} v_L \, dt = \frac{1}{L} \cdot \int_{0}^{t} v_L \, dt + i_L(0)$ (initial current)

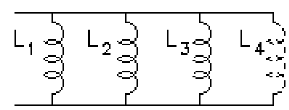
Inductor current **cannot** change instantaneously

Energy stored in electric field: $\quad W_L = \frac{1}{2} \cdot L \cdot I_L^{\ 2}$

series: $\quad L_{eq} = L_1 + L_2 + L_3 + \ldots$



parallel: $\quad L_{eq} = \dfrac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots}$

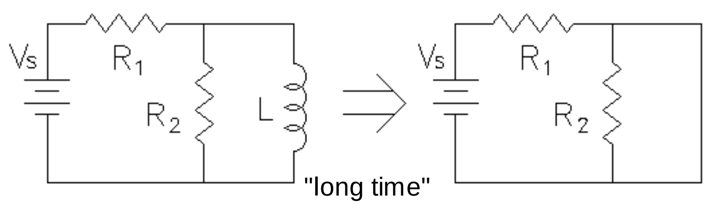


Steady-state of Final conditions

If a circuit has been connected for "a long time", then it has reached a steady state condition. that means the currents and voltages are no longer changing.

$\frac{d}{dt} i_L = 0 \qquad v_L = L \cdot \frac{d}{dt} i_L = 0$

no voltage means it looks like a short



$i_L(\infty) = \frac{V_S}{R_1}$

13. The following circuit has been connected as shown for a long time.

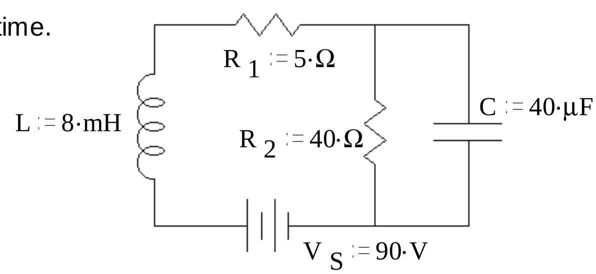
    The energy stored in the inductor is:

    (A) $0.016 \cdot J$

    (B) $0.020 \cdot J$

    (C) $1.3 \cdot J$

    (D) $1.6 \cdot J$



14. The energy stored in the capacitor is:

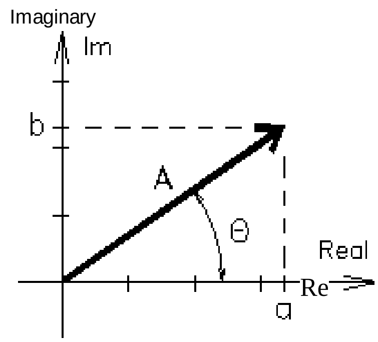
    (A) $0.002 \cdot J$

    (B) $0.041 \cdot J$

    (C) $0.128 \cdot J$

    (D) $0.162 \cdot J$

Complex Numbers



$j = \sqrt{-1}$ the imaginary number

Rectangular Form $\mathbf{A} = a + b\cdot j$

$Re(\mathbf{A}) = a \qquad Im(\mathbf{A}) = b$

Polar Form $\mathbf{A} = A\cdot e^{j\cdot\theta}$

$Re(\mathbf{A}) = A\cdot\cos(\theta) \qquad Im(\mathbf{A}) = A\cdot\sin(\theta)$

Conversions $A = |\mathbf{A}| = \sqrt{a^2 + b^2} \qquad \theta = \arg(\mathbf{A}) = \operatorname{atan}\left(\frac{b}{a}\right)$

$a = A\cdot\cos(\theta) \qquad b = A\cdot\sin(\theta)$

$\mathbf{A} = A\cdot e^{j\cdot\theta} = A\cdot\cos(\theta) + A\cdot\sin(\theta)\cdot j \qquad \mathbf{A} = a + b\cdot j = \left(\sqrt{a^2+b^2}\right)\cdot e^{j\cdot\operatorname{atan}\left(\frac{b}{a}\right)}$

Special Cases $j := \sqrt{-1} = e^{j\cdot 90\cdot deg} \qquad \frac{1}{j} = -j = e^{-j\cdot 90\cdot deg} \qquad e^{j\cdot 0\cdot deg} = 1 \qquad e^{-j\cdot 180\cdot deg} = e^{-j\cdot 180\cdot deg} = -1$

$j\cdot e^{j\cdot\theta} = e^{j\cdot(\theta + 90\cdot deg)}$

Define a 2nd number: rect: $\mathbf{D} = c + d\cdot j$ polar: $\mathbf{D} = D\cdot e^{j\cdot\phi}$

Equality $\mathbf{A} = \mathbf{D}$ if and only if $a = c$ and $b = d$ OR $A = D$ and $\theta = \phi$

Addition and Subtraction

$\mathbf{A} + \mathbf{D} = (a + b\cdot j) + (c + d\cdot j) = (a + c) + (b + d)\cdot j$

$\mathbf{A} - \mathbf{D} = (a + b\cdot j) - (c + d\cdot j) = (a - c) + (b - d)\cdot j$

Convert polars to rectangular form first

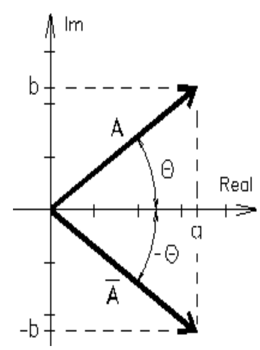
Multiplication and Division $\mathbf{A}\cdot\mathbf{D} = (a + b\cdot j)\cdot(c + d\cdot j) = (a\cdot c - b\cdot d) + (b\cdot c + a\cdot d)\cdot j$

Rectangular: $\frac{\mathbf{A}}{\mathbf{D}} = \frac{a + b\cdot j}{c + d\cdot j} = \frac{a + b\cdot j}{c + d\cdot j}\cdot\frac{c - d\cdot j}{c - d\cdot j} = \frac{a\cdot c + b\cdot d}{c^2 + d^2} + \frac{b\cdot c - a\cdot d}{c^2 + d^2}\cdot j$

Polar: $\mathbf{A}\cdot\mathbf{D} = A\cdot e^{j\cdot\theta}\cdot D\cdot e^{j\cdot\phi} = A\cdot D\cdot e^{j\cdot(\theta + \phi)}$

$\frac{\mathbf{A}}{\mathbf{D}} = \frac{A\cdot e^{j\cdot\theta}}{D\cdot e^{j\cdot\phi}} = \frac{A}{D}\cdot e^{j\cdot(\theta - \phi)}$

Powers $\mathbf{A}^n = A^n\cdot e^{j\cdot n\cdot\theta} = A^n\cdot\cos(n\cdot\theta) + A^n\cdot\sin(n\cdot\theta)\cdot j$ Convert rectangulars first, usually



Conjugates

complex number	Conjugate	
$\mathbf{A} = a + b\cdot j$	$\overline{\mathbf{A}} = a - b\cdot j$	$\overline{\overline{\mathbf{A}}} = \mathbf{A}$
$\mathbf{A} = A\cdot e^{j\cdot\theta}$	$\overline{\mathbf{A}} = A\cdot e^{-j\cdot\theta}$	
$\mathbf{F} = \frac{3 + 5\cdot j}{(2 - 6\cdot j)\cdot e^{j\cdot 40\cdot deg}}$	$\overline{\mathbf{F}} = \frac{3 - 5\cdot j}{(2 + 6\cdot j)\cdot e^{-j\cdot 40\cdot deg}}$	

Euler's equation $e^{j\cdot\alpha} = \cos(\alpha) + j\cdot\sin(\alpha)$ OR: $\cos(\alpha) = \frac{e^{j\cdot\alpha} + e^{-j\cdot\alpha}}{2} \qquad \sin(\alpha) = \frac{e^{j\cdot\alpha} - e^{-j\cdot\alpha}}{2\cdot j}$

$e^{j\cdot(\omega\cdot t + \theta)} = \cos(\omega\cdot t + \theta) + j\cdot\sin(\omega\cdot t + \theta)$

$Re\left[e^{j\cdot(\omega\cdot t + \theta)}\right] = \cos(\omega\cdot t + \theta)$

If we freeze this at time t=0, then we can represent $\cos(\omega\cdot t + \theta)$ by $e^{j\cdot\theta}$

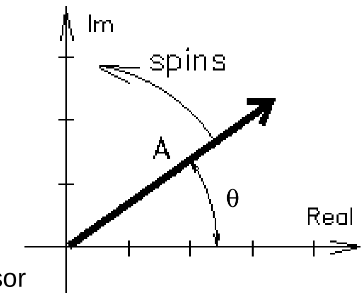
Calculus Remember, when we write $e^{j\cdot\theta}$, we really mean $e^{j\cdot(\omega\cdot t + \theta)}$

$$\frac{d}{dt}\mathbf{A} = \frac{d}{dt}\left(A\cdot e^{j\cdot\theta}\right) = j\cdot\omega\cdot A\cdot e^{j\cdot\theta} = \omega\cdot A\cdot e^{j\cdot(\theta + 90\cdot deg)}$$

$$\int \mathbf{A}\, dt = \int A\cdot e^{j\cdot\theta}\, dt = \frac{1}{j\cdot\omega}\cdot A\cdot e^{j\cdot\theta} = \frac{1}{\omega}\cdot A\cdot e^{j\cdot(\theta - 90\cdot deg)}$$

Phasor analysis of Steady-State Sinusoidal AC

The math is all based on the Euler's equation



Euler's equation $e^{j\cdot\alpha} = \cos(\alpha) + j\cdot\sin(\alpha)$ $\qquad \cos(\alpha) = \dfrac{e^{j\cdot\alpha} + e^{-j\cdot\alpha}}{2}$

OR:

$e^{j\cdot(\omega\cdot t+\theta)} = \cos(\omega\cdot t+\theta) + j\cdot\sin(\omega\cdot t+\theta)$ $\qquad \sin(\theta) = \dfrac{e^{j\cdot\alpha} - e^{-j\cdot\alpha}}{2\cdot j}$

$\mathrm{Re}\left[e^{j\cdot(\omega\cdot t+\theta)}\right] = \cos(\omega\cdot t+\theta)$

If we freeze this at time t=0, then we can represent $\cos(\omega\cdot t+\theta)$ by $e^{j\cdot\theta}$

That's the phasor

<u>Phasor</u>

voltage: $v(t) = V_p\cdot\cos(\omega\cdot t+\phi)$ $\qquad \mathbf{V}(\omega) = V_p\cdot e^{j\cdot\phi}$

current: $i(t) = I_p\cdot\cos(\omega\cdot t+\phi)$ $\qquad \mathbf{I}(\omega) = I_p\cdot e^{j\cdot\phi}$

Phasors are drawn on a complex plane.

Phasors are used for adding and subtracting sinusoidal waveforms.

15. Add the sinusoidal voltages $v_1(t) = 4.5\cdot V\cdot\cos(\omega\cdot t - 30\cdot deg)$

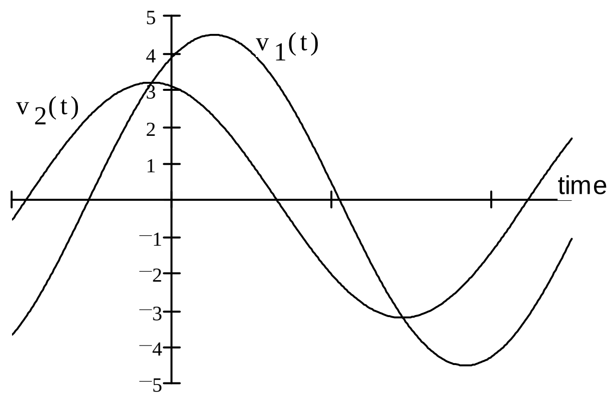
and $v_2(t) = 3.2\cdot V\cdot\cos(\omega\cdot t + 15\cdot deg)$

(A) $7.13\cdot\cos(\omega\cdot t - 11.5\cdot deg)\cdot V$

(B) $7.13\cdot\cos(\omega\cdot t - 45\cdot deg)\cdot V$

(C) $7.7\cdot\cos(\omega\cdot t - 11.5\cdot deg)\cdot V$

(D) $7.7\cdot\cos(\omega\cdot t - 15\cdot deg)\cdot V$



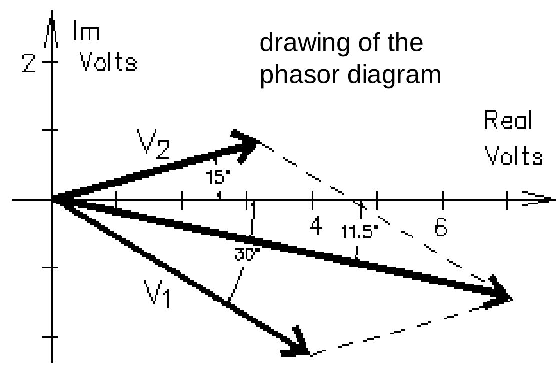
using phasor notation, draw a phasor diagram of the three phasors, then convert back to time domain form.

$v_1(t) = 4.5\cdot V\cdot\cos(\omega\cdot t - 30\cdot deg)$

$\mathbf{V_1}(\omega) = 4.5V\ \underline{/-30^o}$ $\qquad$ or: $\mathbf{V_1}(\omega) = 4.5\cdot V\cdot e^{-j\cdot 30\cdot deg}$

and $v_2(t) = 3.2\cdot V\cdot\cos(\omega\cdot t + 15\cdot deg)$

$\mathbf{V_2}(\omega) = 3.2V\ \underline{/15^o}$ $\qquad$ or: $\mathbf{V_2}(\omega) = 3.2\cdot V\cdot e^{j\cdot 15\cdot deg}$



I'm going to drop the (ω) notation from the phasor notation, it gets cumbersome, but remember that phasors are in the frequency domain..

$\mathbf{V_1} = 4.5V\ \underline{/-30^o}$ $\qquad$ or: $\mathbf{V_1} := 4.5\cdot V\cdot e^{-j\cdot 30\cdot deg}$

$\mathbf{V_2} = 3.2V\ \underline{/15^o}$ $\qquad$ or: $\mathbf{V_2} := 3.2\cdot V\cdot e^{j\cdot 15\cdot deg}$

Add like vectors, first change to the rectangular form

$\mathbf{V_1} = 4.5V\ \underline{/-30^o}$	$4.5\cdot V\cdot\cos(-30\cdot deg) = 3.897\cdot V$	$4.5\cdot V\cdot\sin(-30\cdot deg) = -2.25\cdot V$	$\mathbf{V_1} = 3.897 - 2.25j\ \cdot V$	\
				} add
$\mathbf{V_2} = 3.2V\ \underline{/15^o}$	$3.2\cdot V\cdot\cos(15\cdot deg) = 3.091\cdot V$	$3.2\cdot V\cdot\sin(15\cdot deg) = 0.828\cdot V$	$\mathbf{V_2} = 3.091 + 0.828j\ \cdot V$	/
	Add real parts:	$3.897 + 3.091 = 6.988$	$\mathbf{V_3} := \mathbf{V_1} + \mathbf{V_2}$	
	Add imaginary parts:	$-2.25 + 0.828 = -1.422$	$\mathbf{V_3} = 6.988 - 1.422j\ \cdot V$	sum

Change $\mathbf{V_3}$ back to polar coordinates:

$\sqrt{6.988^2 + 1.422^2} = 7.131$ $\qquad \mathrm{atan}\left(\dfrac{-1.422}{6.988}\right) = -11.502\cdot deg$

Change $\mathbf{V_3}$ back to the time domain:

$v_3(t) = v_1(t) + v_2(t) = 7.13\cdot\cos(\omega\cdot t - 11.5\cdot deg)\cdot V$ (A)

Impedance (like resistance)

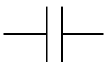
AC impedance

Inductor

$$v_L = L\cdot\frac{d}{dt}i_L \quad = \quad L\cdot\frac{d}{dt}I_p\cdot e^{j\cdot(\omega\cdot t+\theta)} \quad = \quad j\cdot\omega\cdot L\cdot\left[I_p\cdot e^{j\cdot(\omega\cdot t+\theta)}\right]$$

\ in phasor notation ----> $\mathbf{V}_{\mathbf{L}}(\omega) = j\cdot\omega\cdot L\cdot\mathbf{I}(\omega)$ $\qquad \mathbf{Z}_{\mathbf{L}} = j\cdot\omega\cdot L$

Capacitor



$$i_C = C\cdot\frac{d}{dt}v_C \quad = \quad C\cdot\frac{d}{dt}V_p\cdot e^{j\cdot(\omega\cdot t+\theta)} \quad = \quad j\cdot\omega\cdot C\cdot\left[V_p\cdot e^{j\cdot(\omega\cdot t+\theta)}\right]$$

\ in phasor notation ----> $\mathbf{I}_{\mathbf{C}}(\omega) = j\cdot\omega\cdot C\cdot\mathbf{V}(\omega)$

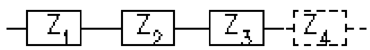
$$\mathbf{V}_{\mathbf{C}}(\omega) = \frac{1}{j\cdot\omega\cdot C}\cdot\mathbf{I}(\omega) \qquad \mathbf{Z}_{\mathbf{C}} = \frac{1}{j\cdot\omega\cdot C} = \frac{-j}{\omega\cdot C}$$

Resistor

$$v_R = i_{R.}\cdot R \qquad \mathbf{V}_{\mathbf{R}}(\omega) = R\cdot\mathbf{I}(\omega) \qquad \mathbf{Z}_{\mathbf{R}} = R$$

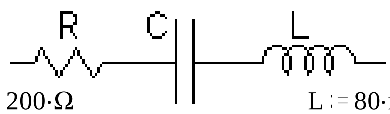
You can use impedances just like resistances as long as you deal with the complex arithmetic.
ALL the DC circuit analysis techniques will work with AC.

series:



$$\mathbf{Z}_{\mathbf{eq}} = \mathbf{Z}_{\mathbf{1}} + \mathbf{Z}_{\mathbf{2}} + \mathbf{Z}_{\mathbf{3}} + \ldots$$

Example:



$f := 500\cdot Hz$ $\qquad R := 200\cdot\Omega$ $\qquad L := 80\cdot mH$

$\omega := 2\cdot\pi\cdot f \quad = \quad \omega = 3141.6\cdot\frac{rad}{sec}$ $\qquad C := 0.6\cdot\mu F$ $\qquad j\cdot\omega\cdot L = 251.327j\cdot\Omega$

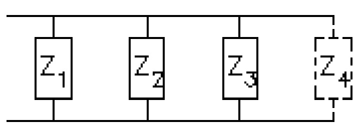
$$\frac{1}{j\cdot\omega\cdot C} = -530.516j\cdot\Omega$$

$$\mathbf{Z}_{\mathbf{eq}} := R + \frac{1}{j\cdot\omega\cdot C} + j\cdot\omega\cdot L \quad = \quad 200\cdot\Omega - 530.5\cdot j\cdot\Omega + 251.3\cdot j\cdot\Omega = 200 - 279.2j\ \cdot\Omega$$ rectangular form

$$\sqrt{(200\cdot\Omega)^2 + (279.2\cdot\Omega)^2} = 343.4\cdot\Omega \qquad \mathrm{atan}\left(\frac{-279.2\cdot\Omega}{200\cdot\Omega}\right) = -54.38\cdot deg$$

$\mathbf{Z}_{\mathbf{eq}} = 343.4\Omega\ \underline{/-54.4}^{o}$ polar form

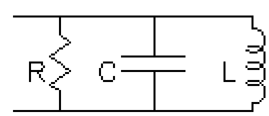
parallel:



$$\mathbf{Z}_{\mathbf{eq}} = \frac{1}{\frac{1}{\mathbf{Z}_{\mathbf{1}}} + \frac{1}{\mathbf{Z}_{\mathbf{2}}} + \frac{1}{\mathbf{Z}_{\mathbf{3}}} + \ldots}$$

Example: Same parts and frequency as above

$f := 500\cdot Hz$ $\qquad \omega := 2\cdot\pi\cdot f$ $\qquad \omega = 3141.6\cdot\frac{rad}{sec}$



$L := 80\cdot mH$

$R := 200\cdot\Omega$ $\qquad C := 0.6\cdot\mu F$ $\qquad \omega\cdot C = 1.885\cdot 10^{-3}\ \cdot\frac{1}{\Omega}$

$$\mathbf{Z}_{\mathbf{eq}} := \frac{1}{\frac{1}{R} + \frac{1}{\left(\frac{1}{j\cdot\omega\cdot C}\right)} + \frac{1}{j\cdot\omega\cdot L}} \quad = \quad \frac{1}{\frac{1}{R} + j\cdot\omega\cdot C - \frac{j}{\omega\cdot L}} \quad = \quad \frac{1}{\frac{1}{200\cdot\Omega} + 1.885\cdot 10^{-3}\cdot j\cdot\frac{1}{\Omega} - 3.979\cdot 10^{-3}\cdot j\cdot\frac{1}{\Omega}}$$

$$= \frac{1}{\left(5\cdot 10^{-3} - 2.094\cdot 10^{-3}\cdot j\right)\cdot\frac{1}{\Omega}}\cdot\frac{5\cdot 10^{-3} + 2.094\cdot 10^{-3}\cdot j}{\left(5\cdot 10^{-3} + 2.094\cdot 10^{-3}\cdot j\right)} = 170.156 + 71.261j\ \cdot\Omega$$

$2.93848\cdot 10^{-5}$

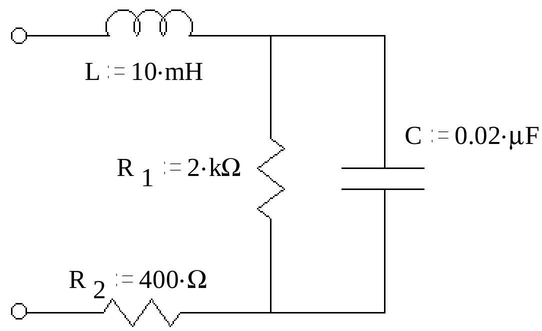
If you want the answer in polar form, it's easier to convert the denominator first.

$$\sqrt{\left(5\cdot 10^{-3}\cdot\frac{1}{\Omega}\right)^2 + \left(2.094\cdot 10^{-3}\cdot\frac{1}{\Omega}\right)^2} = 5.4\cdot 10^{-3}\ \cdot\frac{1}{\Omega} \qquad \mathrm{atan}\left(\frac{2.094\cdot 10^{-3}\cdot\Omega}{5\cdot 10^{-3}\cdot\Omega}\right) = 22.72\cdot deg$$

$$\frac{1}{5.4\cdot 10^{-3}\cdot\frac{1}{\Omega}} = 185.185\cdot\Omega$$

$\mathbf{Z}_{\mathbf{eq}} = 185.2\ \underline{/22.7}^{o}$

16. Find $\mathbf{Z_{eq}}$. $f := 3 \cdot \mathrm{kHz}$



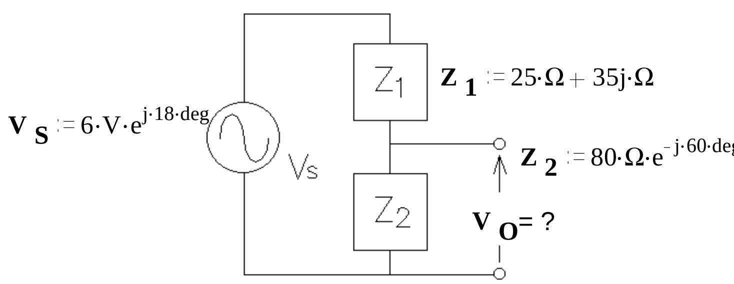
(A) $\mathbf{Z_{eq}} = 442 \underline{/ -25.2^o} \ \Omega$

(B) $\mathbf{Z_{eq}} = 2.03 \underline{/ 34.5^o} \ k\Omega$

(C) $\mathbf{Z_{eq}} = 1.84 \underline{/ -24.8^o} \ k\Omega$

(D) $\mathbf{Z_{eq}} = 3.44 \underline{/ -45.8^o} \ k\Omega$

17. Find $\mathbf{V_O}$ in the circuit shown.



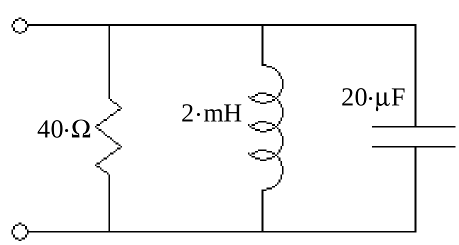
(A) $3.91 \cdot V \cdot e^{j \cdot 19.9 \cdot deg}$

(B) $3.91 \cdot V \cdot e^{-j \cdot 100 \cdot deg}$

(C) $6.53 \cdot V \cdot e^{-j \cdot 69.8 \cdot deg}$

(D) $6.53 \cdot V \cdot e^{-j \cdot 14.2 \cdot deg}$

18. The resonant frequency of the circuit shown is most nearly:



(A) 5 rad/s

(B) 20 rad/s

(C) 5000 rad/s

(D) 20000 rad/s

19. What is the magnitude of the equivalent impedance of the circuit at the resonant frequency?

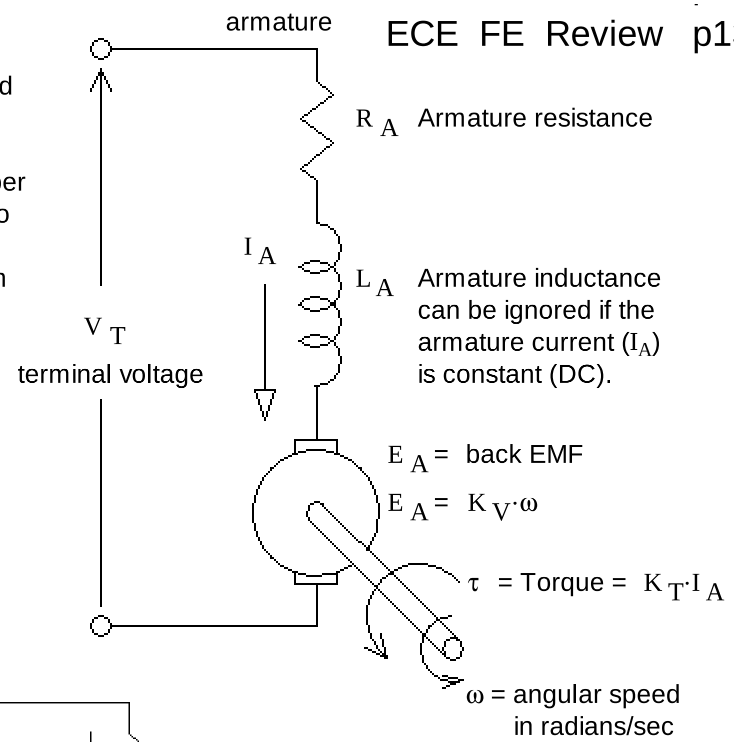
(A) $0 \cdot \Omega$

(B) $4.44 \cdot \Omega$

(C) $4.961 \cdot \Omega$

(D) $40 \cdot \Omega$

DC Motor

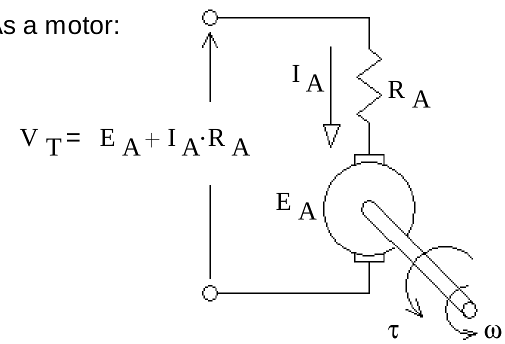


A simple DC motor consists of a rotating armature placed in a stationary magnetic field. The DC current flows through brushes which make contact with a commutator on the rotating armature. The commentator has a number of contact points and is wired to the armature windings so that it's magnetic field is out of alignment with the stationary field. As the motor armature turns to align with the magnetic field, the connection is changed so that the next winding gets the current and the armature field is again out of alignment. The motor torque (τ) is directly proportional to the armature current (I_A) by the torque constant (K_T).

Because the armature windings are turning within a magnetic field, a voltage is induced on those windings. This voltage is known as the back EMF and opposes the armature current flow. The back EMF (E_A) is directly proportional to the armature angular velocity (ω) by the voltage constant (K_V). An electrical model of a DC motor is shown at right.

The stationary field may be created by permanent magnets or by a field winding.

As a motor:



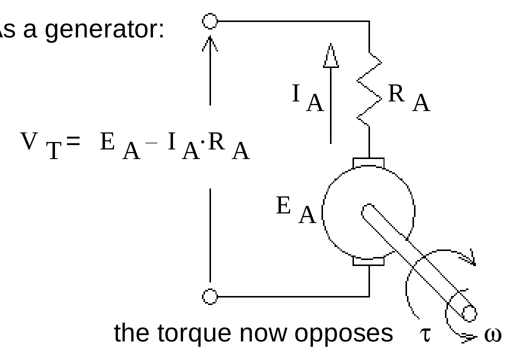
K_T = Torque constant

K_V = Voltage constant

$K_T = K_V$

If the shaft is turned faster by some external driver, the generated EMF could be larger than the terminal voltage. In that case the current reverses direction and the motor becomes a DC generator

As a generator:



the torque now opposes the mechanical driver

20. A DC motor is producing a torque of $30\ \mathrm{Nm}$. The current of the armature is halved. What is the approximate new torque? Ignore the armature resistance.

    (A) $15 \cdot \mathrm{N \cdot m}$

    (B) $20 \cdot \mathrm{N \cdot m}$

    (C) $30 \cdot \mathrm{N \cdot m}$

    (D) $35 \cdot \mathrm{N \cdot m}$

21. A DC motor operates from 24 V (V_T) and has an armature resistance of 0.3Ω. At full load, the armature current is 10 A and the speed is 1200 rpm. Ignoring all losses except the armature resistance, find the no-load speed of the motor. Note: you may assume $I_A = 0$ when the motor is not loaded.

    (A) $1200 \cdot \mathrm{rpm}$

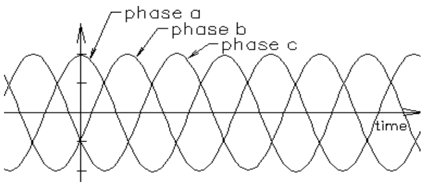
    (B) $1370 \cdot \mathrm{rpm}$

    (C) $1200 \cdot \mathrm{rpm}$

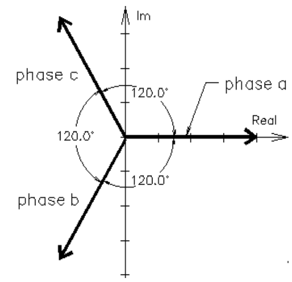
    (D) $2400 \cdot \mathrm{rpm}$

Synchronous Generators & Motors

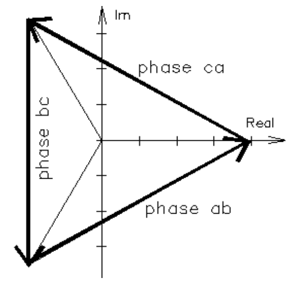
Synchronous machines run on and generate 3-phase power



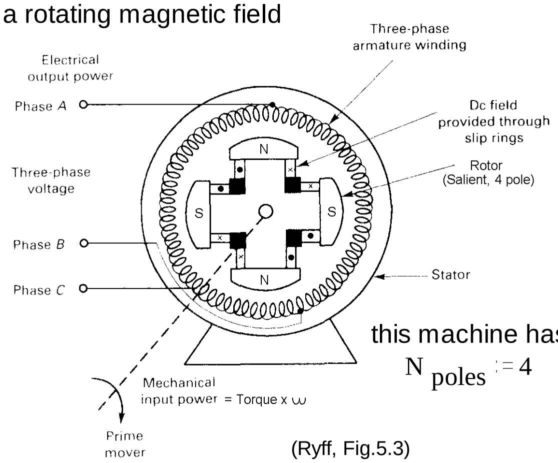
Wye connection:



Delta connection:



The 3 phases are wired in the stator to produce a rotating magnetic field



(Ryff, Fig.5.3)

this machine has $N_{poles} := 4$

for 60Hz systems

$$\text{rotational speed} = n_m \text{ or } n_{sync} = \frac{f \cdot \frac{2 \cdot poles}{cyc} \cdot \frac{60 \cdot sec}{min}}{N_{poles}} = \frac{7200 \cdot rpm}{poles}$$

$$\omega = \frac{4 \cdot \pi \cdot f}{N_{poles}}$$

n_{sync} = the synchronous speed

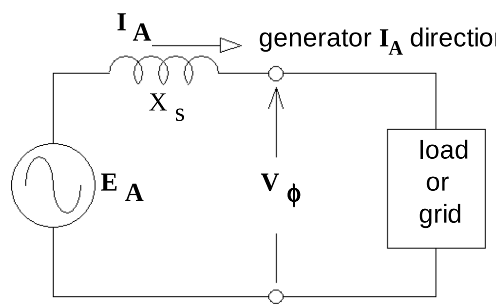
The rotor is an electronmagnet with a DC field current. It will rotate along with the rotating magnetic field at the synchronous speed.

When spinning, the induced armature voltages ($\mathbf{E_A}$ for our phase) depends on the field current, I_f . I_f causes the field flux (called **excitation**).

Electrical analysis on a per-phase basis

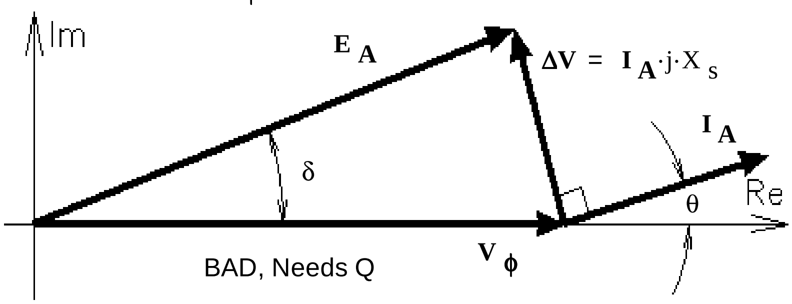
The electrical model of an armature winding:

$\mathbf{X_s}$ is the armature inductance
(armature windings and leakage)
(magnetization)

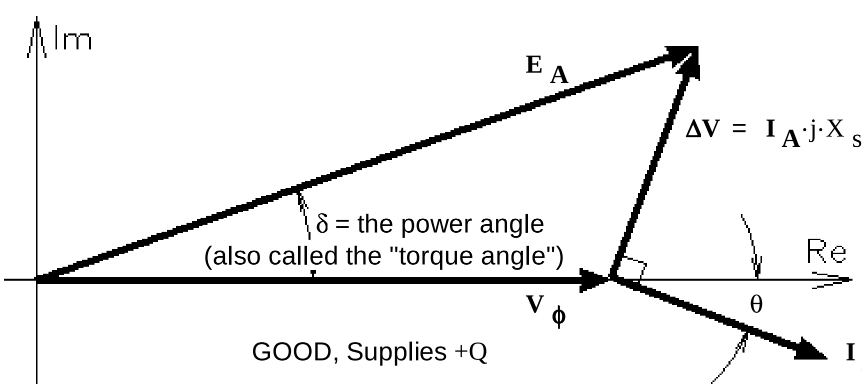


Synchronous Generators

$$\mathbf{E_A} = \mathbf{I_A} \cdot j \cdot X_s + \mathbf{V_\phi}$$



The **under-excited** condition, the current leads the terminal voltage, $\mathbf{V_\phi}$. The generator supplies -Q (-VARs), that is, it absorbs +Q (+VARs), just like an inductive load. Usually not desirable.



$$\mathbf{I_A} = \frac{\mathbf{\Delta V}}{j \cdot X_s}$$

The **over-excited** condition, the current lags the terminal voltage, $\mathbf{V_\phi}$. The generator supplies +Q (+VARs), that is, it absorbs -Q (-VARs), just like an capacitive load. Usually desirable.

Note: if δ reaches 90°, the generator will lose synchronization.